

DEPARTMENT OF BUILDING ACOUSTICS
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THEORY AND EXPERIMENTS
ON SOUND PROPAGATION
ABOVE AN IMPEDANCE BOUNDARY

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FOREWORD

The contents of this report are taken from a thesis on outdoor sound propagation. I wish to express my gratitude to colleagues who have contributed with practical and mental support. Especially I want to thank Prof. Sven G. Lindblad for his genuine interest in all aspects of this work, Mr. Bengt Bengtsson and Mr. Lars Persson for their performance of the experiments, and Miss Monika Johnsson for her most patient help with various aspects on the manuscript.

Lund in November 1977

Sven-Ingvar Thomasson

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INTRODUCTION

The need for this report arose when two papers^{1,2} were written on outdoor sound propagation with and without screens. Firstly, the theory of these two papers is rather heavy and contains many details of limited interest for people working in this field. Therefore a more brief presentation ought to be of great help. This is given in Part I, and the material here is essentially taken from Refs. 3 and 4. Secondly, computer programs were constructed, and in Part II some of the most useful subroutines are published. Finally, together with the theoretical work, many experiments were made. In Part III and Part IV more details are given about the experiments, and also a great many experimental results, which were omitted in the two original papers.

This report is thus more or less an appendix to Refs. 1 and 2. This is the reason why very little is referred to other studies. The references are given in the original papers.

PART I

T H E O R Y

SECTION 1

SOME POSSIBLE MODELS FOR THE PHYSICAL BEHAVIOUR OF GROUNDS

The physical mechanism behind the acoustical properties of the ground is not well understood. This is not surprising. For instance, it is difficult to get hold of which part of a grassland — the straws, the root system or deeper regions — that acoustically play the most important rôle. Further, when choosing a model for the ground, it is most desirable that we obtain a point impedance description as result, as this certainly will make theoretical as well as experimental aspects easier. The semi-infinite space in Fig. 1 is a possibility, but presumably the layer in Fig. 2 on a hard backing is a better choice; the root system

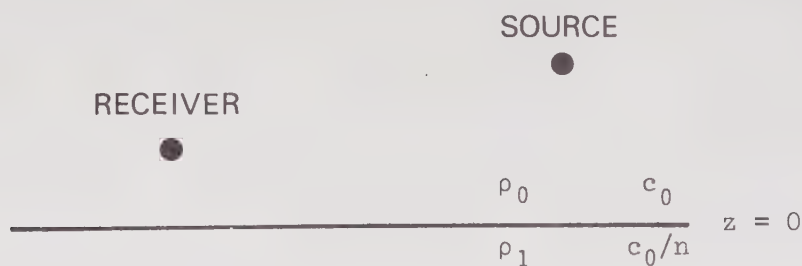


FIGURE 1. The semi-infinite ground model. The density and sound velocity of air ($z > 0$) is ρ_0 and c_0 respectively and of the ground ($z < 0$) ρ_1 and c_0/n respectively. n is the refraction index.

is probably "backed up" by an unpenetrable medium. Also this model, when applicable, makes it possible to scale grounds by a suitable layer, e.g. a felt. Multiple layers and even more sophisticated models may of course also be considered (see e.g. Wait⁵), but as we will see later, the single layer model will give a sufficient number of degrees of freedom in order to explain experimental results.

As already mentioned, we want this model to behave like a point impedance, i.e. to be locally reacting. Later we will treat the sound field from a point source, and as the field from it is built up of an infinite set of plane waves, it might be instructive to see how a plane wave is

ion to the "extended reaction boundary" [Fig. 2(a)] and to the "impedance boundary" [Fig. 2(b)] differ by a term in the order of magnitude $|n|^{-2}$. This statement holds independently of the source and the receiver locations⁶. Particularly this is surprising when we locate the source (which has a strong reactive near field) and the receiver on the ground, and when we choose the impedance of the ground in such a way that the field is strongly attenuated and thus very sensitive to approximations. But, fortunately, the point impedance description still holds⁷. This means that when $|n|^2 \gg 1$ we have unified two approaches to the problem of sound propagation: the point impedance and the extended reaction approaches. The impedance in the case with a point source is the same as the one for plane waves, Eq. (1).

But this information is useless, unless we measure the physical properties ρ_1 , n and z_1 of the layer, which is impossible as we do not know the exact location of the layer, not even if there is a layer of this kind. This difficulty is avoided by instead regarding the layer as a mathematical description, somehow possible to apply to reality; at least to measured frequency responses from a point source.

To obtain the frequency dependence of n and ρ_1 it is reasonable to assume that the layer has some kind of porous properties. A suitable description would then be the homogeneous layer for which

$$n = c_0 [K \rho_p \Omega (1 + i\sigma(\rho_p \omega)^{-1})]^{1/2} \quad (3a)$$

and

$$\rho_1 = \rho_p [1 + i\sigma(\rho_p \omega)^{-1}] \quad (3b)$$

where

- c_0 — the sound velocity of the air
- K — the effective compressibility of the fluid in the layer
(may be complex)
- Ω — the porosity
- σ — the (specific) flow resistance
- ρ_p — the dynamic air density

Not even this solves our problem — we cannot measure all these physical constants either. To avoid this difficulty an implicit method is chosen: we will compare measured and theoretical frequency sweeps and try to estimate these constants indirectly. As we are only interested in the frequency dependence of v we may combine these physical parameters to only four real constants — a , b , c and d — by introducing

$$a = c_0 |K \rho_p \Omega|^{1/2} \rho_0 / \rho_p \quad (4a)$$

$$b = \arg[(K \rho_p)^{1/2}] \quad (4b)$$

$$c = \sigma / \rho_p \quad (4c)$$

and

$$d = [|K \rho_p \Omega|^{1/2} 2\pi z_1]^{-1} \quad (4d)$$

To get an idea of the meaning of the parameters one may roughly state that a approaches the specific characteristic impedance at the high frequencies, $2b$ is the phase angle of the complex compressibility, c is the flow resistance compared to effective density and d is essentially inversely proportional to the thickness.

The admittance may now be written as

$$v = a(b'/c') \tan(b'c'f/d)/i, \quad (5a)$$

where

$$b' = e^{ib} \quad (5b)$$

$$c' = [1 + ic(2\pi f)^{-1}]^{1/2} ; \quad 0 < \arg(c') < \pi/4 \quad (5c)$$

and where f is the frequency. For the sake of simplicity, a , b , c and d are regarded as independent of the frequency. Models with only three parameters have been tried, one with $b = 0$ and one with $c = 0$. These are much simpler to use due to the less degrees of freedom, but they were difficult to apply to experimental data.

We have now converted the problem with a point source above a porous layer to the one with a point source above an impedance boundary, where the point impedance is given by Eqs. (5a)-(5c). Two problems remain:

- (1) How evaluate the solution to the problem of the impedance boundary?
- (2) How estimate the parameters a , b , c and d of the porous layer, i.e., how estimate the ground impedance?

In Secs. 2 and 3 below these problems are dealt with.

Part IV, Figs. 101-148, see page 46. An example of such an estimate is shown in Fig. 4, in which there are two frequency responses taken directly from Figs. 135 and 136.

- (ii) The second step is to try to obtain a more precise estimate by means of trial and error, or better, least squares fit, which is easy when having access to a computer. An example of this is also shown in Fig. 4. We thus find that the measured response in Fig. 4 to a great extent could be explained by an admittance given by Eq. (5) with $a = 0.25$, $b = 10^0$, $c = 7500$ Hz and $d = 425$ Hz. Additional measurements with other geometries should be made to check both the theory and the accuracy of the estimates.

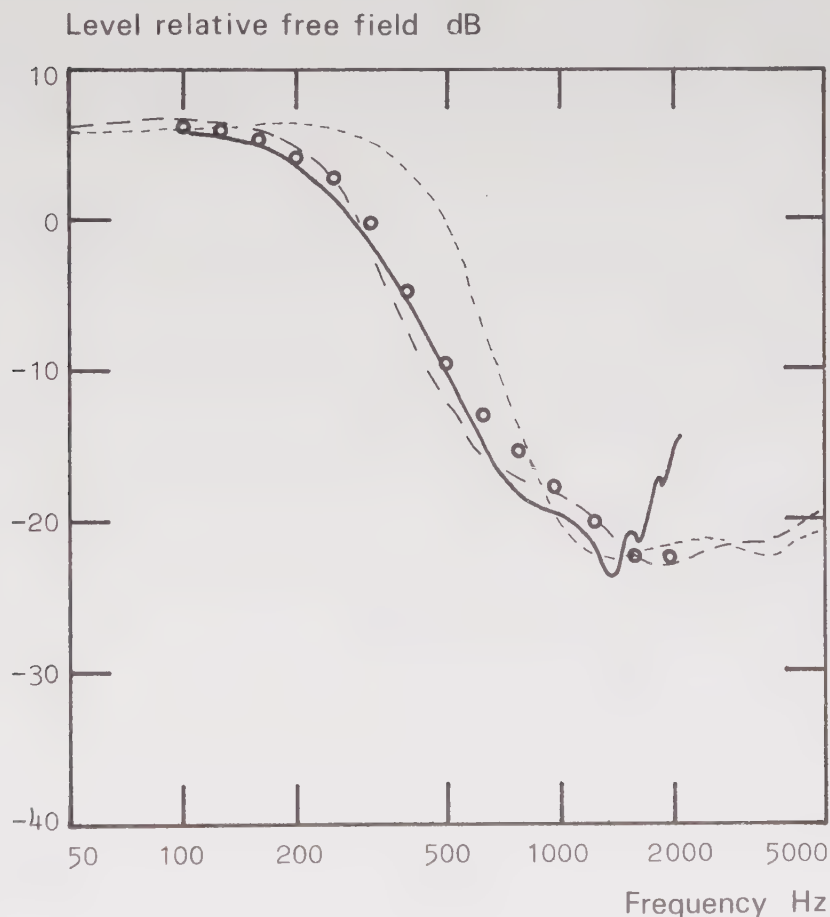


FIGURE 4. Estimation of the impedance. Source height = receiver height = 0.1 m. Distance source-receiver = 20 m.

- Measured, grassland (Fig. 323)
- - - $a = 0.25$, $b = 10^0$, $c = 5000$ Hz, $d = 400$ Hz (Fig. 135)
- · - · $a = 0.25$, $b = 10^0$, $c = 10000$ Hz, $d = 800$ Hz (Fig. 136)
- o o o $a = 0.25$, $b = 10^0$, $c = 7500$ Hz, $d = 425$ Hz (computer).

The method has some disadvantages. It is unusable if the ground does not have the porous properties assumed. We are dependent on having favourable weather conditions regarding wind and temperature gradients, and on having a large plane surface. An additional trouble may be that the real source is not a point source, and that the power output from it is not exactly known.

Often it seems as if these disadvantages could be surmountable and in those cases the method offers some advantages, mainly that standard equipment could be used, that the exact location of the ground does not have to be known (as the frequency responses do not critically depend on the source and receiver heights, see e.g. Figs. 322-323), that we obtain a very simple continuous description of the frequency dependence of the admittance and that the method should not be sensible to minor local variations of the admittance. Another advantage is that the method offers a direct check of the desired quantity — the wave field.

SECTION 3

APPROXIMATE SOLUTION OF IMPEDANCE BOUNDARY

Consider a point source $[\delta(\vec{r}_S - \vec{r}_R)]$ located in $\vec{r}_S = (x_S, y_S, z_S)$, a receiver in $\vec{r}_R = (x_R, y_R, z_R)$ and an impedance boundary at $z = 0$, as in Fig. 2(b). The velocity potential (which is proportional to the pressure) $\Psi_L = \Psi_L(\vec{r}_R | \vec{r}_S)$ fulfils the wave equation

$$(\nabla^2 + k_0^2)\Psi_L = 0 \quad \text{in} \quad z_S, z_R > 0 \quad (6a)$$

and the impedance boundary is defined by

$$\partial\Psi_L/\partial z_R + ik_0 v \Psi_L = 0 \quad \text{at} \quad z_R = 0 \quad (6b)$$

where v is the point admittance, Eq. (1), of the boundary.

The exact solution to this problem is easy to obtain by means of the Fourier transform, and different studies of this problem deal with how to rewrite it into a suitable form. The most efficient exact solution is the one suggested by Ingard, which gives the result in terms of a single integral along a steepest descent contour (further comments, see Refs. 3 and 8). With the digital computers of today there is no need for more efficient solutions. However, as we will see in the next section, where screen on the ground is treated, there is need for a still more efficient solution, since Ψ_L has to be integrated over the screen. Here our model of ground impedance — the porous layer — is of great help, as approximations in Ψ_L are particularly useful when having some knowledge of the impedance.

In Ref. 1, Sec. V, it is shown that under the following two conditions (which are reasonable in view of the practical application of the porous layer)

$$|\gamma_0 - 1| \ll |\gamma_1 - 1| \quad (7a)$$

and

$$k_0 R_2 \gg 1 \quad (7b)$$

a solution of high accuracy is:

$$\Psi_L(\vec{r}_R | \vec{r}_S) = \Psi_I + \Psi_H + \Psi_1 + \Psi_2, \quad (8)$$

where

$$\Psi_I = -(4\pi R_1)^{-1} \exp(ik_0 R_1), \quad (9a)$$

$$\Psi_H = -(4\pi R_2)^{-1} \exp(ik_0 R_2) \quad (9b)$$

and

$$\Psi_1 = \frac{Ck_0 v}{2\pi^{\frac{1}{2}} B} \exp(ik_0 R_2 \gamma_0) \operatorname{erfc}(A), \quad (9c)$$

where $\operatorname{erfc}(A)$ is the complementary error function. For $|A|^2 < 10$ we use the expansion

$$\operatorname{erfc}(A) = 1 - \frac{2A}{\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(-A^2)^m}{m!(2m+1)}, \quad (10a)$$

and for $|A|^2 \geq 10$ the asymptotical series

$$\operatorname{erfc}(A) \sim \frac{\exp(-A^2)}{A\pi^{\frac{1}{2}}} \sum_{m=0}^{\infty} \frac{(2m)!}{m!(-4A^2)^m}. \quad (10b)$$

Further we have defined

$$\Psi_2 = \begin{cases} \Psi_3 & ; \operatorname{Im}(v) < 0; \operatorname{Re}(\gamma_0) > 1, \\ 0 & ; \text{otherwise,} \end{cases} \quad (11a)$$

and

$$\Psi_3 = k_0 v [\pi^{\frac{1}{2}} B]^{-1} \exp(ik_0 R_2 \gamma_0), \quad (11b)$$

with the constants

$$A = [ik_0 R_2 (\gamma_0 - 1)]^{1/2}, \quad (12a)$$

$$B = [ik_0 R_2 (1 - \gamma_1)]^{1/2}, \quad (12b)$$

$$C = \begin{cases} -1 & ; \quad \text{Im}(v) < 0; \text{Re}(\gamma_0) > 1 \\ 1 & ; \quad \text{otherwise} \end{cases} \quad (12c)$$

$$R_1 = [r^2 + (z_S - z_R)^2]^{1/2}, \quad (13a)$$

$$R_2 = [r^2 + (z_S + z_R)^2]^{1/2}, \quad (13b)$$

$$r = [(x_S - x_R)^2 + (y_S - y_R)^2]^{1/2}, \quad (14a)$$

$$\cos(\theta_0) = (z_S + z_R)/R_2 \quad ; \quad 0 \leq \theta_0 < \pi/2 \quad (14b)$$

and

$$\gamma_0 = -v \cos(\theta_0) + (1 - v^2)^{1/2} \sin(\theta_0), \quad (15a)$$

$$\gamma_1 = -v \cos(\theta_0) - (1 - v^2)^{1/2} \sin(\theta_0). \quad (15b)$$

All complex roots, $W^{1/2}$, are defined as $-\frac{1}{2}\pi \leq \arg(W^{1/2}) < \frac{1}{2}\pi$. This branch (or possibly the principal branch) is normally chosen by the computer (in FORTRAN: CSQRT(), see Part II).

This solution, which is sufficiently accurate for our purposes, is numerically very efficient. It is presented as a subroutine (name: FIELD) in Part II, and Table I gives all the parameters of this subroutine. See also Part III, where the solution is compared with experiments.

S E C T I O N 4

A SCREEN ON AN IMPEDANCE BOUNDARY

A. The free field screen

Let us first consider a finite free field screen as in Fig. 5. The

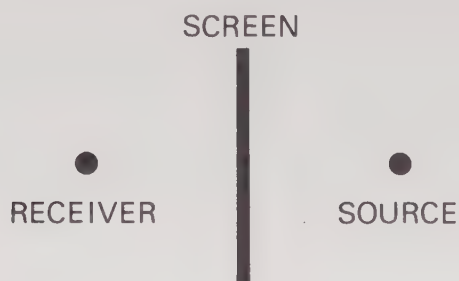


FIGURE 5. The free field screen. The receiver is in $\vec{r}_R = (x_R, y_R, z_R)$ and the source in $\vec{r}_S = (x_S, y_S, z_S)$.

incident field (without screen) Ψ_I is affected by the screen, and the solution may be written as a sum of an incident and a diffracted field:

$$\Psi = \Psi_I + \Psi_{\text{Diff}}. \quad (16)$$

To obtain the exact expression for the diffracted field is usually difficult, and in many cases impossible. One way to achieve a solution is by imposing suitable boundary conditions on the screen, S , and by solving

$$\Psi = \Psi_I + \iint_S (\Psi^R \partial_n \Psi - \Psi \partial_n \Psi^R) dS \quad (17)$$

on S . Ψ^R is a free field solution with a point source at $\vec{r}_R$ and ∂_n is the normal derivative taken with respect to integration variables. The problem is that Ψ is unknown on the screen. For plane obstacles one may overcome this difficulty by giving a solution in the sense of "physical optics" (sometimes called a "Rayleigh solution"). There are theoretical

objections against it, and we will refer to it by a superscript "P.O.". Nevertheless there have been many fruitful applications of it, and later we shall show that the screen on ground is one. A physical solution to the problem Eq. (17) is obtained by putting

$$\begin{pmatrix} \Psi \\ \partial_n \Psi \end{pmatrix} = \begin{cases} \begin{pmatrix} 2\Psi^S \\ 2\partial_n \Psi^S \end{pmatrix} & \text{on the illuminated} \\ & \text{side of the screen} \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \text{on the shadowed} \\ & \text{side of the screen} \end{cases} \quad (18)$$

where the upper case refers to a hard screen ($\partial_n \Psi = 0$ on S) and the lower case to a soft screen ($\Psi = 0$ on S). Ψ^S is the free field solution with a point source at $\vec{r}_S$. Hence the physical optics solution to Eq. (17) may be written as

$$\Psi^{\text{P.O.}} = \Psi_I \begin{cases} -2 \iint_S \Psi^S \partial_n \Psi^R dS & \text{(hard screen)} \\ +2 \iint_S \Psi^R \partial_n \Psi^S dS & \text{(soft screen)} \end{cases} \quad (19a)$$

$$(19b)$$

where the integral is now taken over the illuminated side of the screen only; we have converted the implicit integral equation into an explicit solution.

B. The screen on ground

When, apart from the screen, one has a ground present too, there is the problem with the boundary condition at $z = 0$. Without screen this is taken into account by an image source [at $(x_S, y_S, -z_S)$], which gives a reflected field $\Psi_H + \Psi_1 + \Psi_2$ added to the direct field Ψ_I [see Eqs. (8) and (9)]. In the same manner the condition at $z = 0$ may be taken into account when the screen is present too, Fig. 6, by simply adding the corresponding reflected fields both to Ψ_I and Ψ^R in Eq. (17). Roughly speaking this means that we have reflected not only the direct wave-field, Ψ_I , from the source, but also the scattered wavefield from the "sources" in the integral Eq. (17); some kind of "sources" on the screen. This results in the physical optics solution [cf. Eq. (19)]

$$\psi^{\text{P.O.}} = \psi_L \begin{cases} -2 \iint_S \psi_L^S \partial_n \psi_L^R dS & (\text{hard screen}) \\ +2 \iint_S \psi_L^R \partial_n \psi_L^S dS & (\text{soft screen}) \end{cases} \quad (20a)$$

Here ψ_L , ψ_L^S and ψ_L^R are defined as for the free field screen, but in addition they contain a reflected wavefield from the ground. If we specify the screen geometry according to Fig. 7 this solution is explicitly⁹:

$$\psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \psi_L + \psi_{\text{Hard}}^{\text{Diff}} \quad (21a)$$

$$\psi_{\text{Hard}}^{\text{Diff}} = 2 \int_0^{Z_1} \int_{Y_0}^{Y_1} \psi_L(\vec{r}_0 | \vec{r}_S) \partial \psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R dy dz \quad (21b)$$

for the hard screen and

$$\psi_{\text{Soft}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \psi_L + \psi_{\text{Soft}}^{\text{Diff}} \quad (22a)$$

$$\psi_{\text{Soft}}^{\text{Diff}} = -2 \int_0^{Z_1} \int_{Y_0}^{Y_1} \psi_L(\vec{r}_R | \vec{r}_0) \partial \psi_L(\vec{r}_0 | \vec{r}_S) / \partial x_S dy dz \quad (22b)$$

for the soft screen. $\vec{r}_0$ is on the screen and is given by

$$\vec{r}_0 = (0, y, z) \quad (23)$$

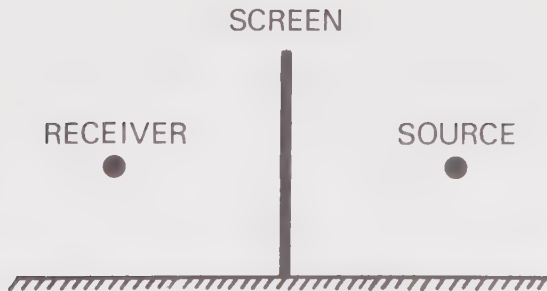


FIGURE 6. The screen on ground.

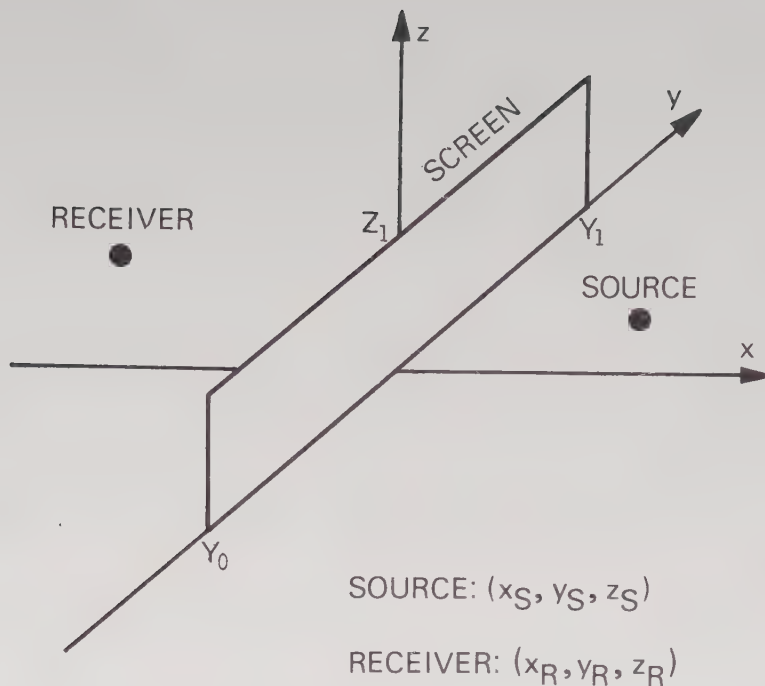


FIGURE 7. Screen geometry.

Thus we have solved the mathematical side of the problem, or at least given an explicit solution. Before going into the numerical side of it, we shall add some comments to this solution¹⁰.

- (i) It cannot separate the effect of a hard and soft screen, since in that case it also violates the reciprocity condition. Reciprocity is violated to the same amount as they differ. Thus if different values of $\Psi_{\text{Hard}}^{\text{P.O.}}$ and $\Psi_{\text{Soft}}^{\text{P.O.}}$ are obtained, say a difference of more than one or two dB, these results should be interpreted with care.
- (ii) A type of Babinet's principle holds also for the screen on ground: We may integrate over the aperture instead. E.g. for a hard screen with $-Y_0 = Y_1 = \infty$ we have, cf. Eq. (21),

$$\Psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = -2 \int_{Z_1}^{\infty} \int_{-\infty}^{\infty} \Psi_L(\vec{r}_0 | \vec{r}_S) \partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R \, dy dz \quad (24)$$

- (iii) The validity of the physical optics solution is unknown. The applicability has to be shown by experiments.

Later we shall show that (i) and (iii) give very few restrictions to the use of the solution; the source and the receiver must be a few wavelengths from the screen, and the solution cannot be used for oblique incidence (for angles larger than 60 degrees in the x,y-plane). These are no real restrictions as to practical applications, in most cases the solution is very accurate, as the experiments in Part III, Sec. 1, will show.

C. Evaluation of the diffracted field

When trying to evaluate the integrals for the diffracted field, Eq. (21b) or (22b), there are essentially two problems arising:

- (1) How evaluate Ψ_L and $\partial\Psi_L/\partial x$?
- (2) How should the numerical integration be made in order to obtain as high efficiency as possible?

Evaluation of Ψ_L

To be able to evaluate Ψ_L it is assumed that the surface could be described in terms of a locally reacting porous layer as in Sec. 1 — we assume that the impedance of the surface could be described by Eq. (5). The impedance is estimated with the method suggested in Sec. 2. This makes it possible to use the approximate solution in Sec. 3 [Eqs. (8)-(15)] to evaluate Ψ_L . (Numerical subroutine: see FIELD, Part II).

Evaluation of $\partial\Psi_L/\partial x$

Numerical differentiation of Ψ_L is not necessary. If the approximations Eqs. (7a) and (7b) from the approximate solution of Ψ_L is used, it is possible to give a very simple expression for $\partial\Psi_L/\partial x$, viz.¹¹

$$\partial\Psi_L(\vec{r}_R|\vec{r}_S)/\partial x_R \cong \partial\Psi_I/\partial x_R + \partial\Psi_H/\partial x_R + G[\Psi_{LR} + \cos(\theta_0)v(\Psi_{LR} + 2\Psi_H)],$$

(25a)

where

$$\partial \Psi_I / \partial x_R = (x_R - x_S)(ik_0 R_1 - 1) \Psi_I / R_1^2, \quad (25b)$$

$$\partial \Psi_H / \partial x_R = (x_R - x_S)(ik_0 R_2 - 1) \Psi_H / R_2^2 \quad (25c)$$

and

$$G = ik_0(x_R - x_S)[R_2 \sin^2(\theta_0)]^{-1}. \quad (25d)$$

This approximate solution of $\partial \Psi_L / \partial x_R$ has the same accuracy as the one of Ψ_L . See further Ref. 2, Sec. IV:B. The numerical evaluation of it has been made at the end of the subroutine, which evaluates Ψ_L (FIELD), see Part II.

The integral

It is far more difficult to see how the integration procedure can be made more efficient. We start from a different point of view by noting that radiation from a plate and diffraction in the sense of geometrical optics are very similar. Consider a vibrating plate (in an infinite baffle) with a travelling wave, as in Fig. 8(a).

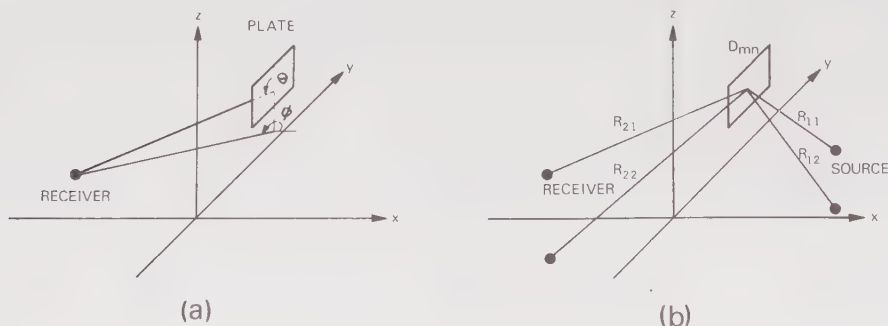


FIGURE 8. The vibrating plate (a) and the segment D_{mn} (b) of the screen.

This gives rise to a farfield of the form

$$\frac{e^{ik_0 R}}{-4\pi R} \cdot \frac{\sin(A)}{A} \cdot \frac{\sin(B)}{B} \cdot C \quad (26)$$

where A, B and C are dependent on the wavenumbers of the plate and the air, the plate dimensions and the angles θ and ϕ [Fig. 8(a)]. Thus $\frac{\sin(A)}{A} \cdot \frac{\sin(B)}{B} \cdot C$ is some kind of angle factor.

Now, reverting to the hard screen on ground, if the screen is very small we may regard the incident wave $\partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R$ in Eq. (21b) as a travelling wave on the plate and evaluate the diffracted field in a similar manner as when obtaining Eq. (26). The diffracted field is then (small screen)

$$\Psi_{\text{Hard}}^{\text{Diff}} \approx \Psi_L(\vec{r}_0 | \vec{r}_S) \partial \Psi_L(\vec{r}_R | \vec{r}_0) / \partial x_R \cdot \frac{\sin(A')}{A'} \cdot \frac{\sin(B')}{B'} \cdot C' \quad (27)$$

where the primed A, B, C's are similar to those for the radiating plate. Unfortunately, screens are not that small. However, it is possible to split up the integral into a number of suitable small segments, D_{mn} , on the screen, as in Fig. 8(b), and perform integration, as in Eq. (27), over each segment separately. Such a procedure will speed up the calculation a factor 10 or more and also, by taking higher order terms into account, it is possible to determine the necessary segment size before the integration is made. This means that it is not necessary to increase the number of integration points until one is sure of the correct value of the integral. Instead one may decide the accuracy of e.g. the insertion loss before the integration procedure is started, and thus take as few segments as possible. Especially when we are satisfied with a result that is accurate within a few dB, the numerical gain is considerable when comparing to e.g. trapezoidal integration or Simpson's rule, see further Ref. 2, Table I.

To be able to do so one relates the relative error ϵ of $\Psi_{\text{Hard}}^{\text{Diff}}$ by

$$\epsilon \cong \begin{cases} 0.11 \cdot \Delta IL \cdot 10^{-IL_M/20} & ; \quad IL_M \geq 10 \text{ dB} \\ 0.11 \cdot \Delta IL \cdot 10^{-0.5} & ; \quad IL_M < 10 \text{ dB} \end{cases} \quad (28a)$$

to an upper limit, IL_M , for the calculated insertion loss, $IL_{\text{Hard}}^{\text{P.O.}}$, and from the error ΔIL in $IL_{\text{Hard}}^{\text{P.O.}}$ (if the calculated result $IL_{\text{Hard}}^{\text{P.O.}}$ is larger than IL_M one either takes IL_M as a conservative estimate of $IL_{\text{Hard}}^{\text{P.O.}}$ or increases IL_M and recalculates $IL_{\text{Hard}}^{\text{P.O.}}$). By investigating the theoretical relative error of $\Psi_{\text{Hard}}^{\text{Diff}}$ it is also possible to relate ϵ to the segment size. This is done by using Eqs. (32) and (33), where now ϵ is known from Eq. (28). In these equations $M \times N$ is the number of segments.

Thus it is possible to determine the segment size, or equivalently, M and N from IL_M and ΔIL by using Eqs. (28), (32) and (33).

We have now expressed the double integral Eq. (21b) as a double sum, see Eq. (29c), with a known error as to the insertion loss. The full result may be expressed as¹² (the soft screen case is obtained by exchanging $[\psi_{1k}^{mn} \partial \psi_{2l}^{mn} / \partial x_R]$ in Eq. (29c) for $[-\psi_{2l}^{mn} \partial \psi_{1k}^{mn} / \partial x_S]$):

$$\psi_{\text{Hard}}^{\text{P.O.}}(\vec{r}_R | \vec{r}_S) = \psi_L + \psi_{\text{Hard}}^{\text{Diff}} \quad (29a)$$

where

$$\psi_{\text{Hard}}^{\text{Diff}} = \sum_{k=1}^2 \sum_{l=1}^2 \psi_{kl}^{\text{Diff}}, \quad (29b)$$

$$\psi_{kl}^{\text{Diff}} = 8 y_D z_D \sum_{m=1}^M \sum_{n=1}^N F \psi_{1k}^{mn} \partial \psi_{2l}^{mn} / \partial x_R. \quad (29c)$$

$$\psi_{11}^{mn} = -(4\pi R_{11})^{-1} \exp(ik_0 R_{11}), \quad (30a)$$

$$\psi_{21}^{mn} = -(4\pi R_{21})^{-1} \exp(ik_0 R_{21}), \quad (30b)$$

$$\psi_{12}^{mn} = \psi_L(0, y_m, z_n | x_S, y_S, z_S) - \psi_{11}^{mn}, \quad (30c)$$

$$\psi_{22}^{mn} = \psi_L(x_R, y_R, z_R | 0, y_m, z_n) - \psi_{21}^{mn}. \quad (30d)$$

$$F = \frac{\sin(k_0 y_D^A)}{k_0 y_D^A} \times \frac{\sin(k_0 z_D^B)}{k_0 z_D^B}, \quad (31a)$$

$$A = \frac{y_m - y_S}{R_{1k}} + \frac{y_m - y_R}{R_{2l}}, \quad (31b)$$

$$B = \frac{z_n - K_k z_S}{R_{1k}} + \frac{z_n - K_l z_R}{R_{2l}}, \quad (31c)$$

$$R_{1k}^2 = x_S^2 + (y_m - y_S)^2 + (z_n - K_L z_S)^2, \quad (31d)$$

$$R_{2L}^2 = x_R^2 + (y_m - y_R)^2 + (z_n - K_L z_R)^2, \quad (31e)$$

$$K_L = \begin{cases} 1 & ; \quad L = 1, \\ -1 & ; \quad L = 2, \end{cases} \quad (31f)$$

and

$$y_D = (Y_1 - Y_0)/(2M), \quad (32a)$$

$$z_D = Z_1/(2N), \quad (32b)$$

$$y_m = (2m - 1)y_D + Y_0, \quad (32c)$$

$$z_n = (2n - 1)z_D. \quad (32d)$$

We also have

$$M = 1 + \text{Int}\left[\frac{1}{2}(Y_1 - Y_0)/Y_{\text{Min}}\right], \quad (33a)$$

$$N = 1 + \text{Int}\left[\frac{1}{2} Z_1/Z_{\text{Min}}\right], \quad (33b)$$

where $\text{Int}[\]$ denotes the integer part,

$$Y_{\text{Min}} = Z_{\text{Min}} = \min[\epsilon X_{\text{Min}}, (\epsilon X_{\text{Min}}/k_0)^{1/2}] \quad (33c)$$

and

$$X_{\text{Min}} = \min(|x_S|, |x_R|). \quad (33d)$$

ϵ is related to IL_M and ΔIL according to Eq. (28).

The only problem left is to decide the largest error acceptable, ΔIL , for the insertion loss, and the largest insertion loss, IL_M , considered. If ΔIL is chosen to 2 dB, then ϵ is 0.07 and 0.02 if IL_M is 10 and 20 dB

respectively [see Eq. (28)]. In many practical cases we know that the insertion loss cannot exceed, say, 10 dB. This indicates that $\epsilon = 0.05$ is sufficient ($\Delta IL \lesssim 1.5$ dB). Then, according to Eqs. (32) and (33), the segments of the screen (used when integrating) are squares with an approximate area of 2.2 m^2 (100 Hz) and 0.2 m^2 (1000 Hz) if neither the source nor the receiver are closer to the screen than 20 m. As the execution time for Ψ_L (subroutine FIELD in Part II) is roughly $0.01 \times$ number of elements, the solution is possible to use in practice.

Note that if the level relative free field with screen, L_{Hard} , is the desired quantity, we should still choose ϵ with respect to the insertion loss, as there are no relations between L_{Hard} and M or N.

S E C T I O N 5

PRACTICAL APPLICATIONS

This report contains essentially three practically useful points: the method for determination of the ground impedance, the solution to the problem of a plane impedance boundary and the physical optics solution to the problem of a screen on the impedance boundary.

A. Ground impedance

To estimate the impedance, which must be known if we are going to proceed to problems with and without screen, it is suggested that the method from Sec. 2 is used. Then the impedance is described in the entire frequency range of interest via four parameters a , b , c and d , together with Eq. (5).

In this context one may use the frequency responses in Part IV, Sec. 1, and the solution to the impedance boundary in Subsec. B below. When the ground impedance is known, through this or any other method, one may proceed to the impedance boundary in general.

B. Propagation above impedance boundary

Here the full solution is given by Eqs. (8)-(15) in Sec. 3. When the admittance, v , and the source and receiver locations ($\vec{r}_S$ and $\vec{r}_R$ respectively) are known, one may use the subroutine FIELD in Part II. In this part program lists and program tests are given and parameters are explained, see Table I.

The level relative free field, which is a natural quantity, is defined as

$$L = 20 \times \log |\psi_L / \psi_I| = 20 \times \log |(P1 + P2) / P1|$$

ψ_L and ψ_I in Eqs. (8) and (9a) correspond to the program variables $P1 + P2$

and P1 respectively. The program calling FIELD must give v , either via Eq. (5) or according to some other method. FIELD also gives the derivatives $\partial \Psi_I / \partial x_R$ and $\partial(\Psi_L - \Psi_I) / \partial x_R$ (correspond to program variables P1X and P2X).

C. Screen on impedance boundary

The physical optics solution to this problem is given by Eqs. (28)-(33), and in addition one needs the solution to the impedance boundary, Eqs. (8)-(15) and its derivatives, Eq. (25). The two latter have already been treated; they are possible to obtain from the subroutine FIELD. The integration procedure in Eqs. (28)-(33) is given in the subroutine INTEG, which apart from FIELD also uses FUNC1, see Part II. The input is, as before, source and receiver positions ($\vec{r}_S$ and $\vec{r}_R$ respectively, where now $x_S > 0$ and $x_R < 0$), the admittance v , the wavenumber k_0 and also screen coordinates, see Fig. 7 and Table I.

The parameter ϵ requires special attention (program variable EPS). This is the relative error for the diffracted field and should be chosen according to Eq. (28), where ΔIL is an estimate of the absolute error for the calculated insertion loss, and IL_M is the maximum insertion loss considered. Once ΔIL and IL_M have been chosen, INTEG gives a fixed result as to the diffracted wavefields Ψ_{Hard}^{Diff} and Ψ_{Soft}^{Diff} (correspond to program variables PH and PS respectively) and also a possibility to evaluate the insertion loss

$$IL_{Hard}^{P.O.} = 20 \times \log |\Psi_L / (\Psi_L + \Psi_{Hard}^{Diff})| = 20 \times \log |(P1 + P2) / (P1 + P2 + PH)|$$

and the level relative free field

$$L_{Hard}^{P.O.} = 20 \times \log |(\Psi_L + \Psi_{Hard}^{Diff}) / \Psi_I| = 20 \times \log |(P1 + P2 + PH) / P1|$$

and similarly for the soft screen case (exchange PH for PS). If the calculated result $IL_{Hard}^{P.O.}$ (or $IL_{Soft}^{P.O.}$) is less than IL_M (dB), we may regard the calculated value, $IL_{Hard}^{P.O.}$, correct within ΔIL (dB). If not, one either takes IL_M as a conservative estimate of the insertion loss, or increases IL_M and recalculates $IL_{Hard}^{P.O.}$.

Execution times on a UNIVAC 1108 computer (cycletime, 0.75 μ s): for FIELD around 0.01 seconds and for INTEG in the order of 1 second to 10 seconds, depending on the size of the screen, the frequency, the distance source/receiver to the screen and the desired accuracy.

Finally a few comments on the applicability of the physical optics solution. It is suggested that both $\psi_{\text{Hard}}^{\text{P.O.}}$ and $\psi_{\text{Soft}}^{\text{P.O.}}$ are calculated. Reciprocity is violated to the same amount as $\psi_{\text{Hard}}^{\text{P.O.}}$ and $\psi_{\text{Soft}}^{\text{P.O.}}$ differ. When they differ too much, the solution should be used with care.

Known limitations of the solution are:

- (a) Both the source and the receiver must be a few wavelengths from the screen, so that the approximate solution of ψ_L , Eqs. (8)-(15), is valid.
- (b) The source and the receiver must be "far" away from the aperture ($x = 0$), so that the approximate solution of $\partial\psi_L/\partial x_R$, Eq. (25), is valid.
- (c) Experiments have shown — reciprocity is violated — that we cannot use the solution for grazing incidence [the angle in the x,y-plane between the line source-receiver and the normal to the screen should not exceed, say, 60 degrees, cf. (a) and (b)].

These restrictions are no real practical restrictions. In most cases the solution is very accurate and the experiments of Part III shall confirm this.

PART II

CALCULATION ROUTINES

CALCULATION ROUTINES

This appendix shows some of the FORTRAN SUBROUTINES (FIELD, INTEG and FUNC1, list see page 31) used for evaluation of the field above a plane impedance boundary with and without screen. All computations have been made with the aid of an 1108 UNIVAC digital computer.

In Table I the input/output variables (parameters) of FIELD and INTEG are shown. This information should be sufficient for the user to understand the function of the routines. FIELD calculates Ψ_L according to Eqs. (8)-(15) and Fig. 2(b). INTEG calculates the integrals over the screen ($\Psi_{\text{Hard}}^{\text{Diff}}$ and $\Psi_{\text{Soft}}^{\text{Diff}}$) according to Eqs. (8)-(15), (25), (29)-(33) and Fig. 7. FUNC1 is the weighting function F, Eq. (31a), and is referenced in INTEG.

Suitable checks of the SUBROUTINES are:

```
CALL FIELD(P1,P2,P3,P4,R1,R2,20.0,0.0,1.0,0.5,5.0,
*(0.5,-0.5))
```

Result:

$$\begin{aligned} P1 &= -0.3491 \cdot 10^{-2} + i \, 0.1906 \cdot 10^{-2} \\ P2 &= 0.3278 \cdot 10^{-2} - i \, 0.1381 \cdot 10^{-2} \\ P3 &= -0.9353 \cdot 10^{-2} - i \, 0.1755 \cdot 10^{-1} \\ P4 &= 0.7113 \cdot 10^{-2} + i \, 0.1631 \cdot 10^{-1} \end{aligned}$$

```
CALL INTEG(P1,P2,10.0,-10.0,10.0,-10.0,0.0,2.0,1.0,
*1.0,0.5,5.0,(0.5,-0.5),0.1)
```

Result:

$$\begin{aligned} P1 &= -0.1862 \cdot 10^{-4} - i \, 0.4579 \cdot 10^{-4} \\ P2 &= -0.2907 \cdot 10^{-4} - i \, 0.4706 \cdot 10^{-4} \end{aligned}$$

It should be easy to write a main program, using FIELD, INTEG and FUNC1, that calculates some desired quantities. As an example, which could be used to check the program, consider a case where the geometry is given by [see Fig. 7]

$$(x_S, y_S, z_S) = (20.0, 1.0, 0.5) \text{ m};$$

$$(x_R, y_R, z_R) = (-20.0, 0.0, 1.0) \text{ m};$$

$$(y_0, y_1, z_1) = (-10.0, 10.0, 1.0) \text{ m};$$

where the impedance is given by [Eq. (5)]

$$a = 0.25; b = 10.0^0; c = 7500.0 \text{ Hz}; d = 425.0 \text{ Hz};$$

and where the frequency equals 500 Hz. ϵ is put equal to 0.02. Then the admittance (v) is

$$v = 0.1586 - i 0.0806.$$

Without screen the total wavefield ($\psi_L = P1 + P2$) is

$$\psi_L = -0.3624 \cdot 10^{-3} + i 0.3211 \cdot 10^{-3}$$

and the level relative free field is

$$L = 20 \times \log |\psi_L / \psi_I| = -12.27 \text{ dB}.$$

With screen the diffracted wavefields $\psi_{\text{Hard}}^{\text{Diff}}$ and $\psi_{\text{Soft}}^{\text{Diff}}$ (equal PH and PS respectively) are

$$\psi_{\text{Hard}}^{\text{Diff}} = 0.7274 \cdot 10^{-4} - i 0.8571 \cdot 10^{-4},$$

$$\psi_{\text{Soft}}^{\text{Diff}} = 0.7436 \cdot 10^{-4} - i 0.8714 \cdot 10^{-4}.$$

The total wavefields $\psi_{\text{Hard}}^{\text{P.O.}}$ and $\psi_{\text{Soft}}^{\text{P.O.}}$ (equal PH + P1 + P2 and PS + P1 + P2 respectively) are

$$\psi_{\text{Hard}}^{\text{P.O.}} = -0.2897 \cdot 10^{-3} + i 0.2354 \cdot 10^{-3},$$

$$\psi_{\text{Soft}}^{\text{P.O.}} = -0.2881 \cdot 10^{-3} + i 0.2339 \cdot 10^{-3}$$

and the insertion loss is

$$IL_{\text{Hard}} = 20 \times \log |\Psi_L / \Psi_{\text{Hard}}^{\text{P.O.}}| = 2.26 \text{ dB},$$

$$IL_{\text{Soft}} = 20 \times \log |\Psi_L / \Psi_{\text{Soft}}^{\text{P.O.}}| = 2.31 \text{ dB}.$$

Finally it is pointed out that when $\Psi_{\text{Hard}}^{\text{Diff}}$ or $\Psi_{\text{Soft}}^{\text{Diff}}$ is evaluated with INTEG, then Ψ_L has to be evaluated with FIELD. If different approximate solutions of Ψ_L are used to evaluate the diffracted field ($\Psi_{\text{Hard}}^{\text{Diff}}$ or $\Psi_{\text{Soft}}^{\text{Diff}}$) and the direct field $\Psi_L(\vec{r}_R | \vec{r}_S)$ (i.e. without screen), then the result $\Psi_{\text{Hard}}^{\text{P.O.}}$ or $\Psi_{\text{Soft}}^{\text{P.O.}}$ is likely to be erroneous. The reason is that $\Psi_{\text{Hard}}^{\text{Diff}}$ is counteracted by $\Psi_L(\vec{r}_R | \vec{r}_S)$ when the insertion loss is large. However, there is no restriction to the type of approximate solution of Ψ_L as long as the same is used throughout the calculation procedure.

FORTRAN SUBROUTINE FIELD

```

SUBROUTINE FIELD(P1,P2,PIX,P2X,R1,R2,X,Y,ZS,ZR,K0,U)
REAL K0
COMPLEX S1,S2,W0,W1,V0,V1,CEX1,CEX2,C0,C1
COMPLEX P1,P2,P3,P4,P5,P6,P7
COMPLEX PE,PE2,PX,PIX,P2X
COMPLEX U,ROOT
ROOT=CSQRT((1.,0.))-U*U)
PIROOT=1.7724539
PI2=6.2831853
PI4=12.5663706
R=X*X+Y*Y
R1=SQRT(R+(ZS-ZR)**2.)
R2=SQRT(R+(ZS+ZR)**2.)
R=SQRT(R)
TETA1=(ZS+ZR)/R2
TETA2=R/R2
AKOR1=K0*R1
AKOR2=K0*R2
S1=AKOR1*(0.,1.)
S2=AKOR2*(0.,1.)
W0=-U*TETA1
W1=ROOT*TETA2
V0=W0+W1-(1.,0.)
V1=W0-W1-(1.,0.)
ARG1=COS(AKOR1)
ARG2=SIN(AKOR1)
CEX1=ARG1+(0.,1.)*ARG2
ARG1=COS(AKOR2)
ARG2=SIN(AKOR2)
CEX2=ARG1+(0.,1.)*ARG2
RN1=R1*PI4
RN2=R2*PI4
P1=-CEX1/RN1
P2=-CEX2/RN2
CONST=1.
PE=-V0*S2
C0=CSQRT(-PE)
C1=CSQRT(-V1*S2)
P4=(0.,0.)
P7=PIROOT*C0
PE2=PE*(2.,0.)
ABSPE=CABS(PE)
IF (REAL(C0*C1).GE.0.) GO TO 5
CONST=-1.
P4=(-2.,0.)*P7*CEXP(-PE)
CONTINUE
IF (ABSPE.LT.10.) GO TO 20
P3=(1.,0.)
P5=(1.,0.)
DO 10 I=1,5
AA=2*I-1
P5=AA*P5/PE2
P3=P3+P5
GO TO 30
20 P3=(1.,0.)
P5=P3
AA=1.
AC=0.
ABSP5=1.
N=MAX1(10,0,ABSPE*11.+0.5)
DO 25 I=1,N
AB=AA
AA=AA+2.
AC=AC+1.
AD=AB/(AA*AC)
P5=P5*PE*AD
ABSP5=ABSP5*ABSPE*AD
IF (ABSP5.LT.1.E-6) GO TO 26
P3=P3+P5
P3=(P7+PE2*P3)*CEXP(-PE)
CONS=CONST*K0/PI2
P6=CONS*U*CEX2/(C0*C1)
P3=(P3+P4)*P6
XR12=X/(R1*R1)
XR22=X/(R2*R2)
PIX=P1*(S1-(1.,0.))*XR12
P2X=P2*(S2-(1.,0.))*XR22
FACT=K0*X/(R*TETA2)
PX=FACT*(P3+U*TETA1*(P3+(2.,0.)*P2))*(0.,1.)
P2=P2+P3
P2X=P2X+PX
RETURN
END

```

γ_0 and γ_1
 Ψ_I and Ψ_H
 Ψ_3
Eq. (10b)
Eq. (10a)
Eq. (25)

FORTRAN SUBROUTINES INTEG AND FUNC1

```

SUBROUTINE INTEG(PH,PS,XS,XR,Y1,Y0,YS,YR,
*ZS,Z1,ZR,K0,U,EPS)
COMPLEX P11,P12,P21,P22,P11X,P12X,P21X,P22X
COMPLEX PH,PS,U
REAL K0
PH=(0.,0.)
PS=(0.,0.)
AM=AMIN1(EPS*ABS(XS),SQRT(EPS*ABS(XS)/K0))
BM=AMIN1(EPS*ABS(XR),SQRT(EPS*ABS(XR)/K0))
AM=AMIN1(AM,BM)
AM=AM*2.
BM=AM
IA=IFIX((Y1-Y0)/AM+1.)
IB=IFIX(Z1/BM+1.)
AM=(Y1-Y0)/FLOAT(IA)
BM=Z1/FLOAT(IB)
AMK=AM*K0/2.
BMK=BM*K0/2.
DO 20 NIA=1,IA
DO 20 NIB=1,IB
YINT=FLOAT(NIA)*AM-AM/2.+Y0
ZINT=FLOAT(NIB)*BM-BM/2.
YSINT=YS-YINT
YRINT=YR-YINT
CALL FIELDP(P11,P12,P11X,P12X,R11,R12,XS,YSINT,ZS,ZINT,K0,U)
CALL FIELDP(P21,P22,P21X,P22X,R21,R22,XR,YRINT,ZR,ZINT,K0,U)
CALL FUNC1(AMK,YSINT,YRINT,R11,R21,YSI11)
CALL FUNC1(AMK,YSINT,YRINT,R11,R22,YSI12)
CALL FUNC1(AMK,YSINT,YRINT,R12,R21,YSI21)
CALL FUNC1(AMK,YSINT,YRINT,R12,R22,YSI22)
CALL FUNC1(BMK,ZINT-ZS,ZINT-ZR,R11,R21,ZSI11)
CALL FUNC1(BMK,ZINT-ZS,ZINT+ZR,R11,R22,ZSI12)
CALL FUNC1(BMK,ZINT+ZS,ZINT-ZR,R12,R21,ZSI21)
CALL FUNC1(BMK,ZINT+ZS,ZINT+ZR,R12,R22,ZSI22)
YSI11=YSI11*ZSI11
YSI12=YSI12*ZSI12
YSI21=YSI21*ZSI21
YSI22=YSI22*ZSI22
PH=PH+P11*P21X*YSI11
PH=PH+P11*P22X*YSI12
PH=PH+P12*P21X*YSI21
PH=PH+P12*P22X*YSI22
PS=PS-P11X*P21*YSI11
PS=PS-P11X*P22*YSI12
PS=PS-P12X*P21*YSI21
PS=PS-P12X*P22*YSI22
CONTINUE
ABM=AM*BM*2.
PH=PH*ABM
PS=PS*ABM
RETURN
END

```

20

```

SUBROUTINE FUNC1(A,B,D,C,E,F)
G=A*(B/C+D/E)
F=1.
IF(ABS(G).GT.0.000001) F=SIN(G)/G
RETURN
END

```


TABLE I

OUTPUT VARIABLES			INPUT VARIABLES		
Variable	Type	Corresponds to	Variable	Type	Corresponds to
P1	COMPLEX	ψ_I	X	REAL	$x_R - x_S$
P2	COMPLEX	$\psi_H + \psi_1 + \psi_2$	Y	REAL	$y_R - y_S$
P1X	COMPLEX	$\partial \psi_I / \partial x_R$	ZS	REAL	z_S
P2X	COMPLEX	$\partial (\psi_H + \psi_1 + \psi_2) / \partial x_R$	ZR	REAL	z_R
R1	REAL	$(x^2 + y^2 + (z_S - z_R)^2)^{1/2}$	KØ	REAL	k_0
R2	REAL	$(x^2 + y^2 + (z_S + z_R)^2)^{1/2}$	U	COMPLEX	v
PH	COMPLEX	ψ_{Hard}^{Diff}	XS	REAL	x_S
PS	COMPLEX	ψ_{Soft}^{Diff}	XR	REAL	x_R
			Y1	REAL	y_1
			YØ	REAL	y_0
			YS	REAL	y_S
			YR	REAL	y_R
			ZS	REAL	z_S
			Z1	REAL	z_1
			ZR	REAL	z_R
			KØ	REAL	k_0
			U	COMPLEX	v
			EPS	REAL	ϵ
FIELD					
INTEG					

PART III

EXPERIMENTS

SECTION 1

SCALE MODEL EXPERIMENTS

Outdoor sound propagation is often influenced by effects as wind and temperature gradients — factors that are important, but also difficult to involve in the models. Further we do not know how to describe outdoor surfaces and especially if there is a layer of the kind described in the previous sections and, in such a case, the exact location of it. To be able to check the theory more in detail and see if the approximations done are relevant, we have to avoid these complications. Therefore a model experiment was carried out in an anechoic chamber.

A. Without screen — the ground

The ground was simulated by a felt of 0.003 m thickness and covered a $3.3 \times 3.2 \text{ m}^2$ acoustically hard backing. A jet noise source was used, the acoustical centre of which was thought to be located 0.01 m above the surface, see Plate 1. The receiver (a $1/4''$ microphone) was placed in

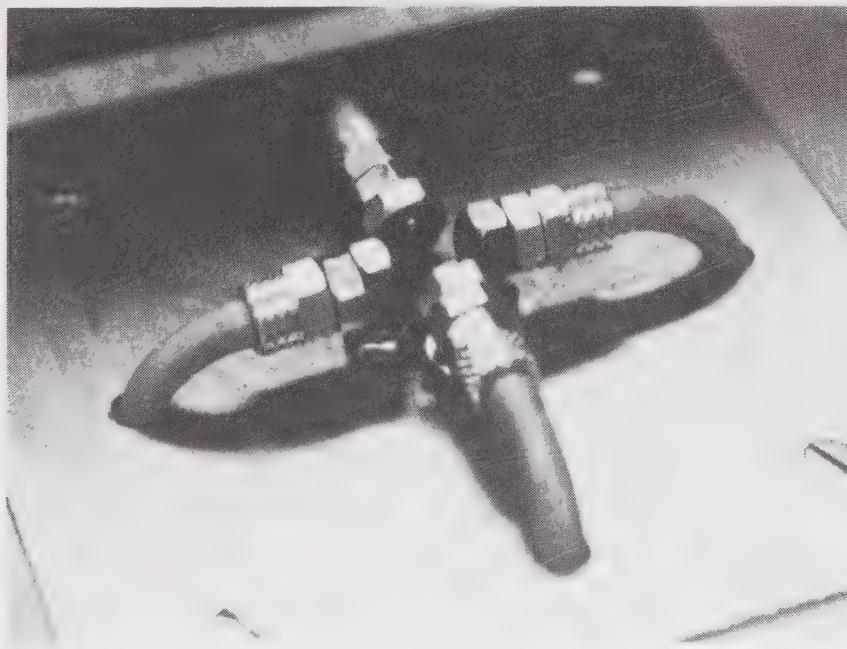


PLATE 1. The jet noise source.

some positions to estimate the ground impedance and illustrate the theory. The frequency response was studied in the range 0.5-50 kHz and was obtained by making a measurement without felt, i.e. with a perfectly hard boundary ($\nu = 0$), which gave, compared to theoretical responses for the hard boundary, a possibility to determine the free field level — taken as 0 dB.

Different aspects of this type of ground are given in Ref. 1, Sec. VIII. Here we omit this figure material and instead give a brief summary. It was found that

- (i) the layer indeed behaved like a point impedance in the sense that measured and theoretical responses agreed well for one single choice of the parameters a , b , c and d of ν ;
- (ii) the layer is not homogeneous. This was the conclusion that could be drawn from an experiment with a layer of twice the thickness compared to one felt (two felts were used). Two entirely different sets of the parameters a , b , c and d had to be used for the single and the double layer. This indicates that the model is applicable to inhomogeneous layers too (evidently there is a sufficient number of degrees of freedom within the model);
- (iii) surface waves were experimentally verified by showing that the so called surface wave term, Ψ_3 , Eq. (11b), could dominate the field. This wave has other properties than a spherical wave.

B. With screen

The screens were made of aluminium plates (1 mm thick). In a preliminary investigation it was found that in the range 0.5-5 kHz vibrations in the plate, or possibly other phenomena, affected the diffraction. To avoid these problems the aluminium plate was replaced by a 14 mm thick screen (12 mm wood, on each side covered with 1 mm aluminium plates) in the range 0.5-5 kHz, see Plate 2.

Much of the experimental material has been presented previously. To show some of the features of the field behind a screen, most of the material

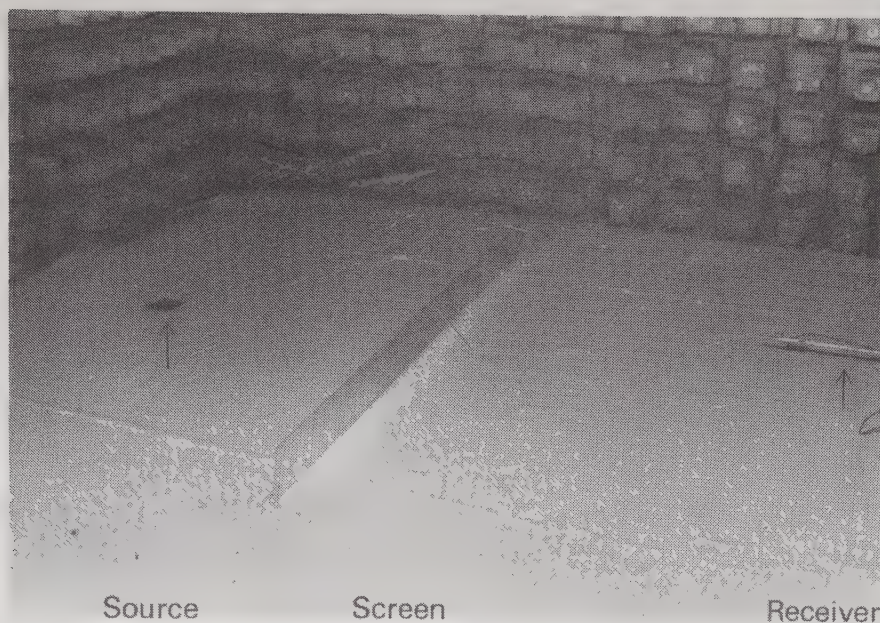


PLATE 2. The experimental set-up for the scale model experiment.

is presented here, see Figs. 201-222 (Part IV). Some interesting aspects of sound propagation above a layer with a screen are given in Ref. 2, viz. the influence of ground impedance (Ref. 2, Fig. 6 or Fig. 16. Note the need for taking the ground properties into account), receiver height (Ref. 2, Fig. 7; see also Figs. 201-222 below), screen height (Ref. 2, Fig. 8), screen length (Ref. 2, Fig. 10), side edge position (Ref. 2, Fig. 11), angle of incidence (Ref. 2, Fig. 12), other types of barriers (Ref. 2, Fig. 13) and also a detailed aspect of the diffraction: diffraction around the side edges (Ref. 2, Fig. 9).

Taken together, the scale model comparisons between experimental and theoretical data, both with and without screens, give credit both to the theory of the locally reacting porous layer (Part I: Sec. 1), the method of measuring the ground impedance (Part I: Sec. 2) and the physical optics solution (Part I: Sec. 4). The agreement is generally excellent.

SECTION 2

OUTDOOR EXPERIMENTS

The measurements were carried out in the late summer and at night to prevent the wind from disturbing them. But although it was perfectly calm during all measurements, the days before had been extremely warm, and it is possible that the measurements have been affected by temperature gradients. No attempt was made to measure the meteorological conditions, and our theory is not capable of taking them into account.

The source was a small loudspeaker fed by a sliding pure tone from a generator. The generator was connected to a slave-filter through which the signal from the microphone passed. All levels in the outdoor measurements have been related to the free field level obtained from measurements with the loudspeaker in an anechoic chamber, and this level has been taken as 0 dB. It may of course be discussed, whether it is proper to correct the loudspeaker output in this way, as the power output may be affected by the reflexes from the surface and therefore considered as partly unknown, but it is probably the best way at this stage.

A remarkable effect in the measurements is the dip at 1250 Hz (see e.g. Fig. 323) that appears, more or less pronounced, in all measured responses. In the preliminary outdoor studies problems arose with the electrical ground due to the damp grassland, which led to interference patterns at low levels. The phenomenon was not fully understood, but it is very unlikely that this dip should be due to changes in the ground impedance, as it does not appear in our model experiment and has not been found by other workers. Therefore we consider the measurements valid in the frequency range 0.1-1 kHz only.

A. Without screen — the ground

Measurements were made on a stubble-field that had been ploughed shortly before. The surface was very rough with local variations in height

around 10 cm, but otherwise plane, and the structure seemed very porous.
See Plates 3-4 and Figs. 301-303.



PLATE 3. The stubble-field.



PLATE 4. The stubble-field.

A few responses were also taken from a newly sown field with very scarce vegetation, but as it seemed, with a very porous structure. See Plates 5-6 and Figs. 304-307.



PLATE 5. New-sown.



PLATE 6. New-sown.

Most of the measurements were made on plane grassland (of the type used for sports) to check the theory more in detail. The grass was fairly newcut and the straws were some centimeters high. See Plates 7-8 and Figs. 308-335.



PLATE 7. The grassland.



PLATE 8. The grassland.

The general agreement between theory and experiment in these cases is satisfactory, but not as good as for the scale model experiment. Whether this is due to the meteorological conditions or the ground properties of grassland is difficult to say at the present stage, but further experiments are planned.

B. With screen

The measurements with screen were made three days later under the same meteorological conditions: no wind but temperature inversion. The positions of the source and the receiver were not exactly the same as without screen, but this should not give better agreement than otherwise. The impedance was checked (see Fig. 311) but the limited data did not indicate that the time spread was larger than the spatial spread.

The screen was built up of elements ($1.5 \times 1 \text{ m}^2$ with thickness 0.015 m) of laminated wood, normally used to mould concrete. This choice allows us both to regard the screen as thin and to neglect transmission through it. The screen was 1.5 m high and its length was 25 m. See Plates 9-10.

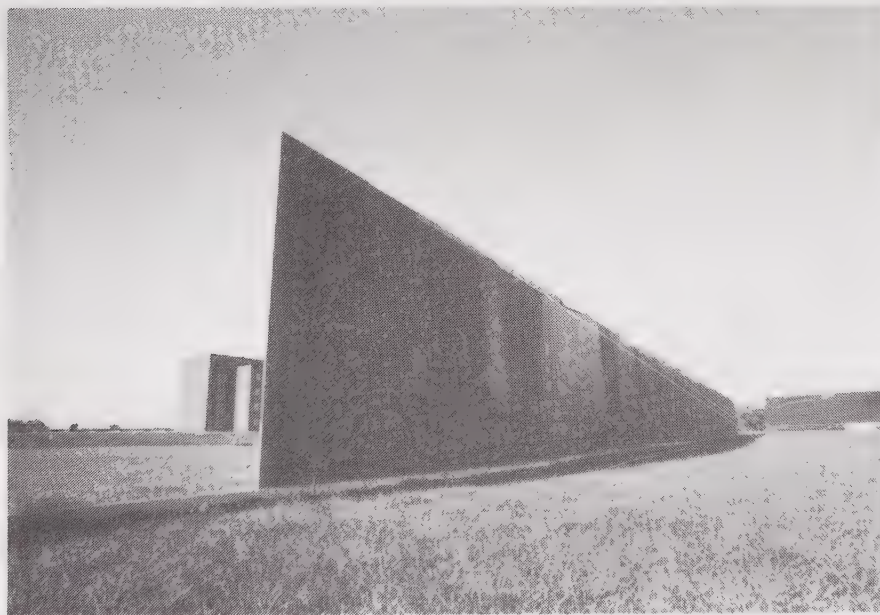


PLATE 9. The screen.



PLATE 10. The screen.

The full experimental data for the screen is presented in Figs. 336-363 (levels relative free field) and Figs. 364-379 (insertion loss). By comparing these two sets of figures one immediately sees that it is much easier to predict the insertion loss than the levels relative free field (both those with and without screen). Whether this can be explained in terms of the meteorological influence (which could have a similar effect on the field both with and without screen) or the ground impedance (which does not critically affect the insertion loss, see Ref. 2, Fig. 16) is not known. As without screen, the outdoor experiments do not give as good agreement theory-experiment as the indoor experiments, but it is satisfactory.

PART IV

FIGURES

S E C T I O N . 1

GROUND IMPEDANCE RESPONSES

The following figures, 101-148, may be used to get a coarse estimate of the parameters a , b (degrees), c (Hz) and d (Hz) of v , see Part I: Sec. 2. They have been calculated with Eqs. (5) and (8)-(15), for different sets of a , b , c and d , with a horizontal distance source-receiver $r = [(x_S - x_R)^2 + (y_S - y_R)^2]^{1/2} = 20$ m, and with source and receiver heights $z_R = z_S = 0.1$ m, see Fig. 2. The subroutine FIELD (see Part II) has been used for the evaluation. The responses are relative free field, i.e. the vertical axis is

$$L = 20 \times \log |\Psi_L / \Psi_I|.$$

They may also be used for any geometry $r = 20.0 \times l$ and $z_R = z_S = 0.1 \times l$. Then c , d and f (frequency) must be scaled $1/l$.

Note that increasing a corresponds to increasing characteristic impedance at the high frequencies, increasing b to increasing phase angle for the complex compressibility, increasing c to increasing flow resistance (or decreasing effective density) and increasing d essentially to decreasing thickness.

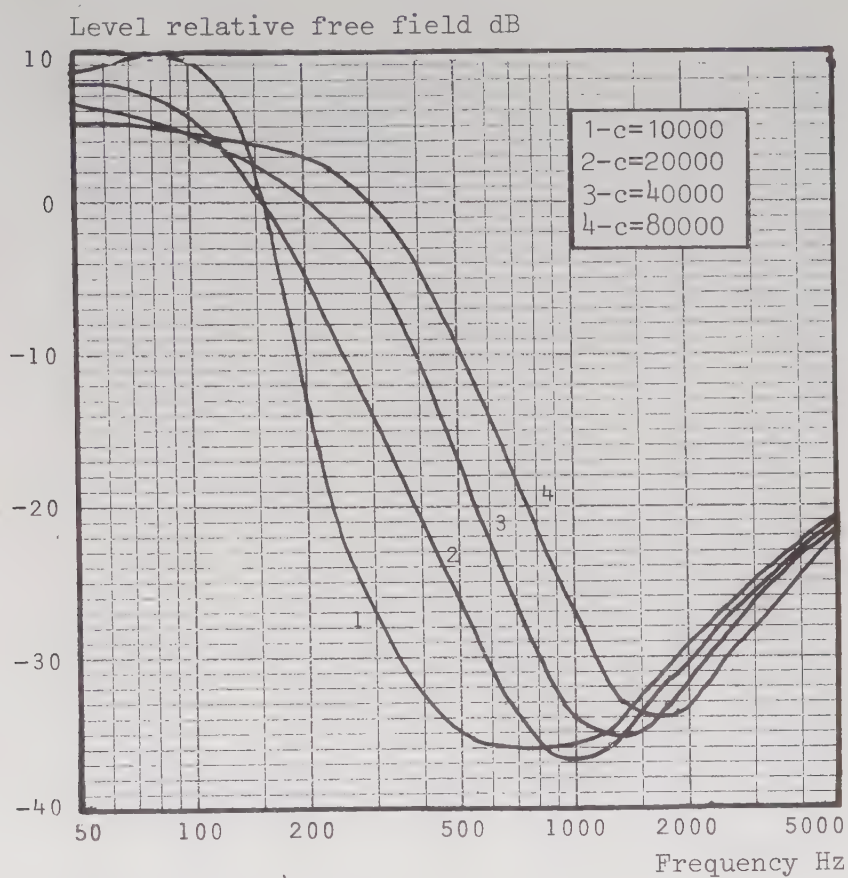


FIG.101: d = 400

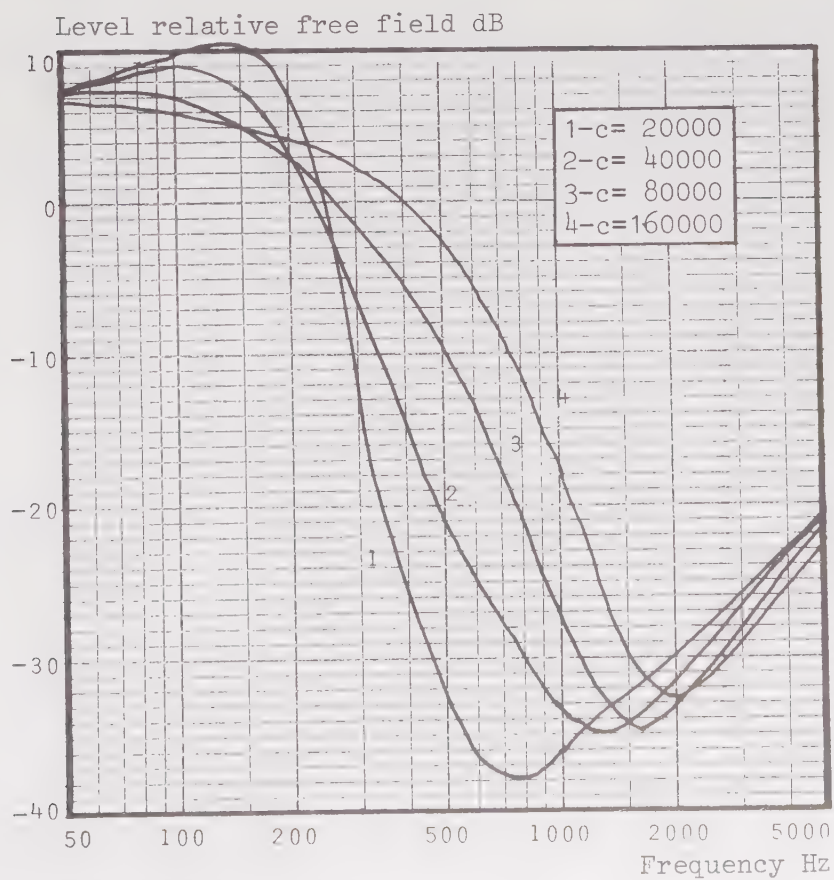
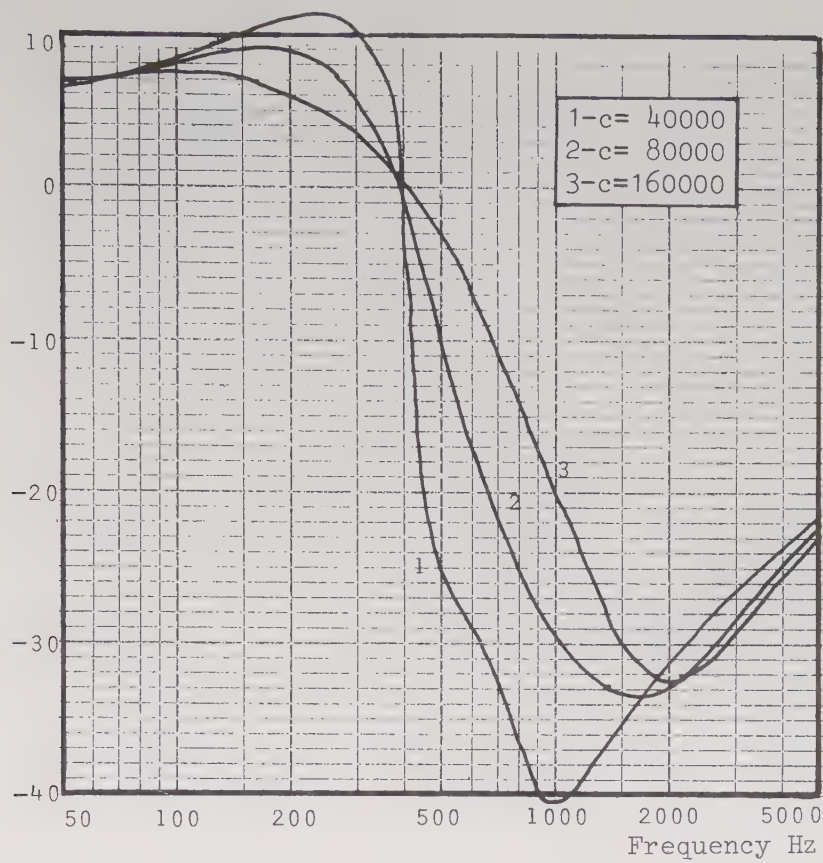
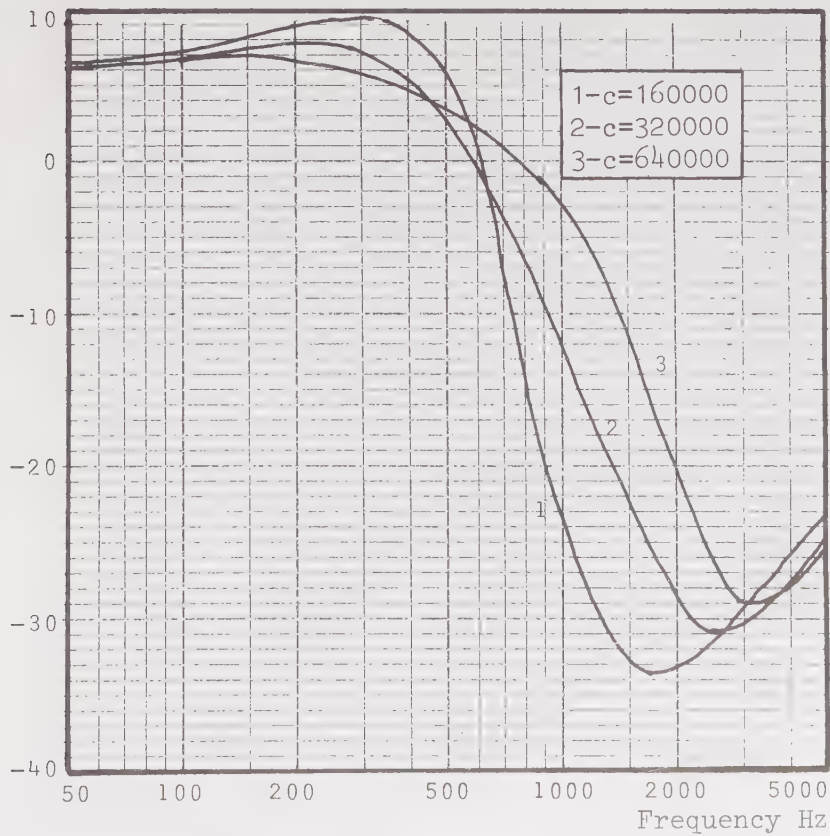


FIG. 102: d = 800

Level relative free field dB

FIG. 103: $d = 1600$

Level relative free field dB

FIG. 104: $d = 3200$

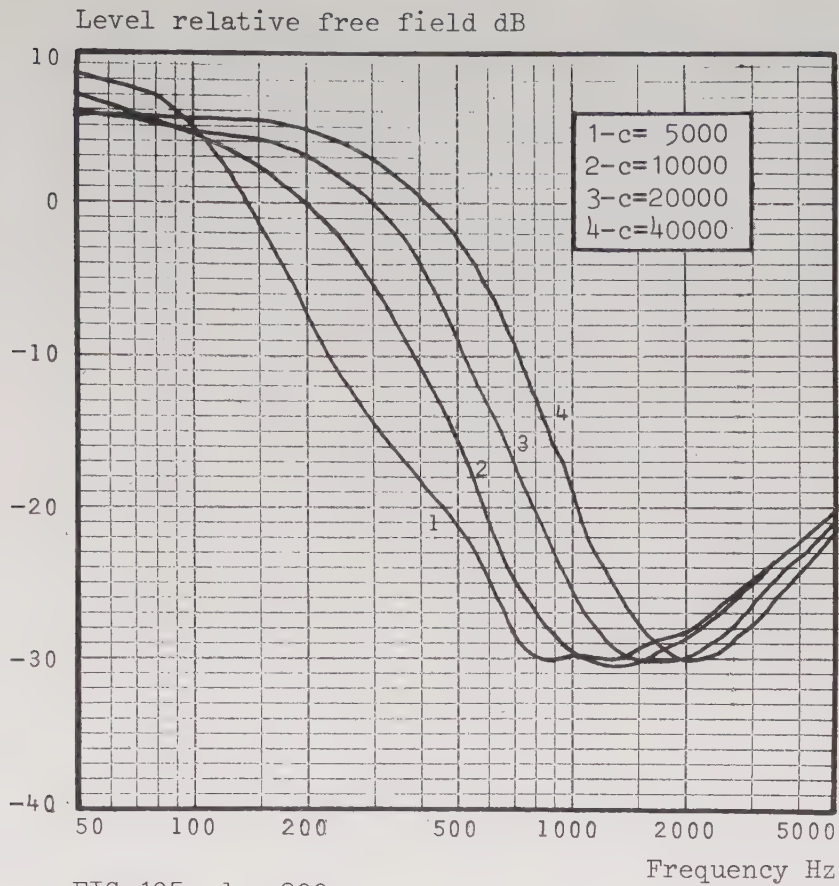


FIG.105: $d = 200$

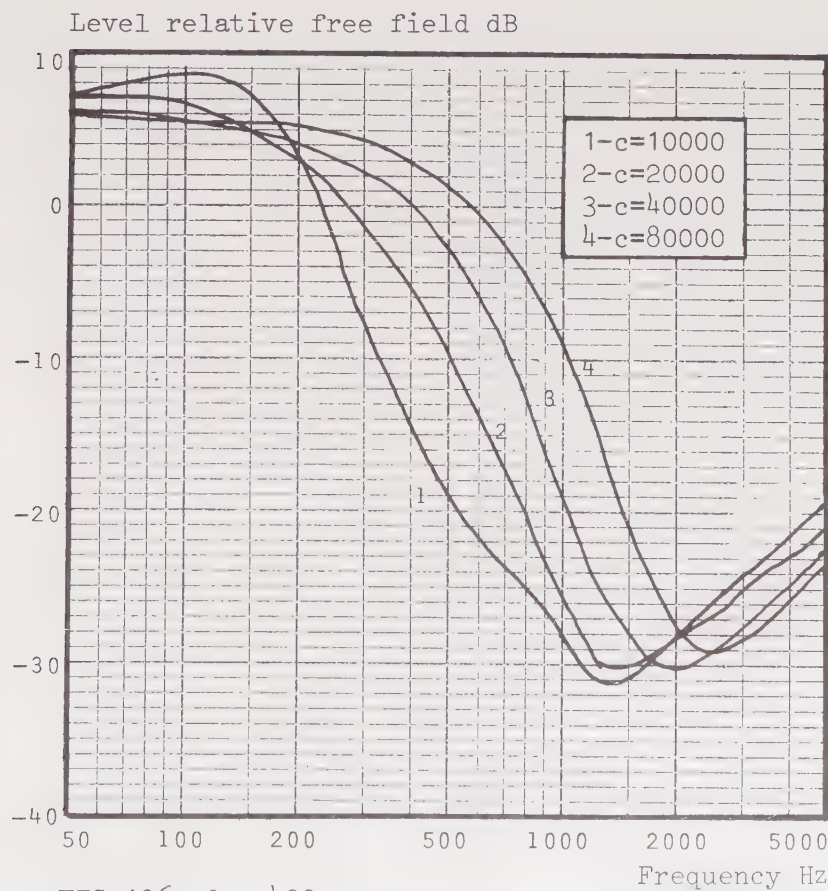


FIG.106: $d = 400$

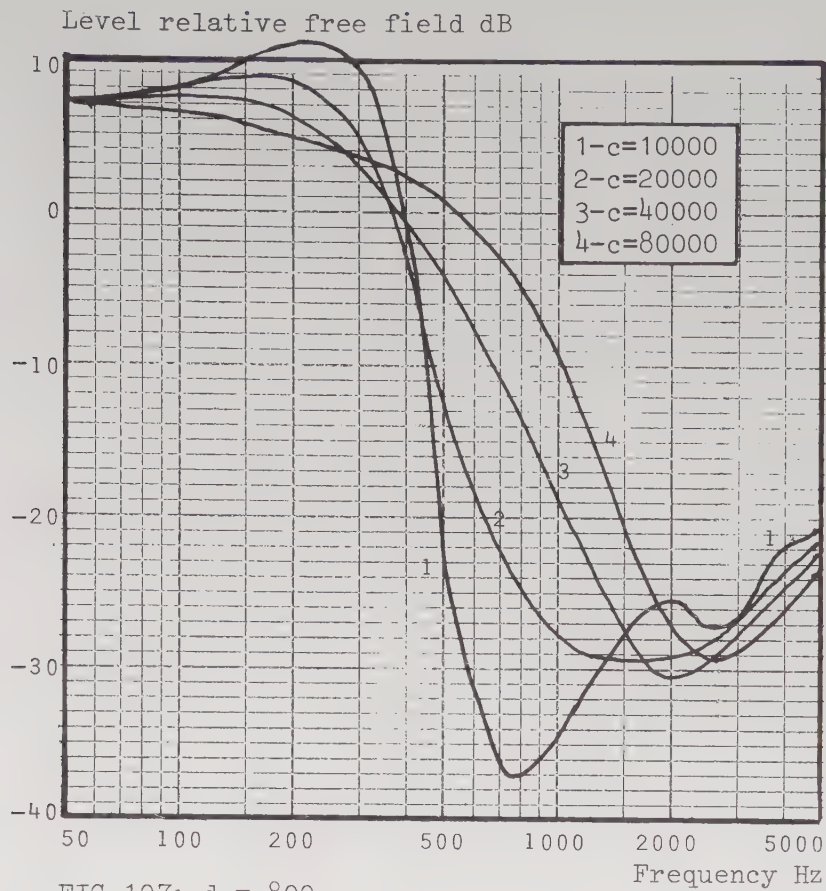


FIG.107: d = 800

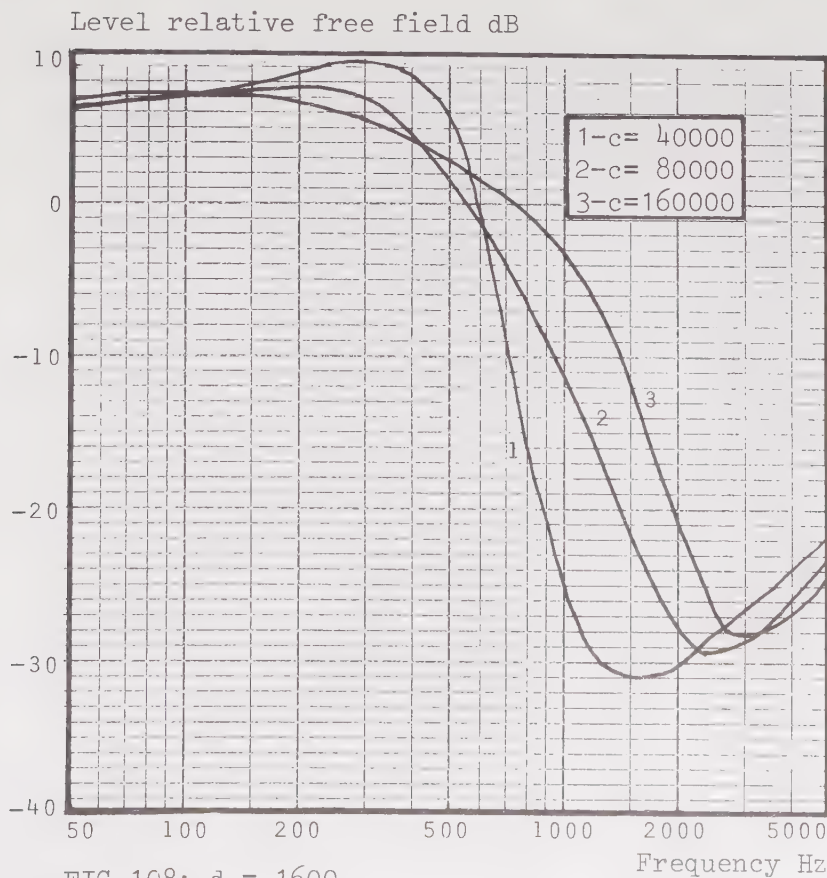


FIG.108: d = 1600

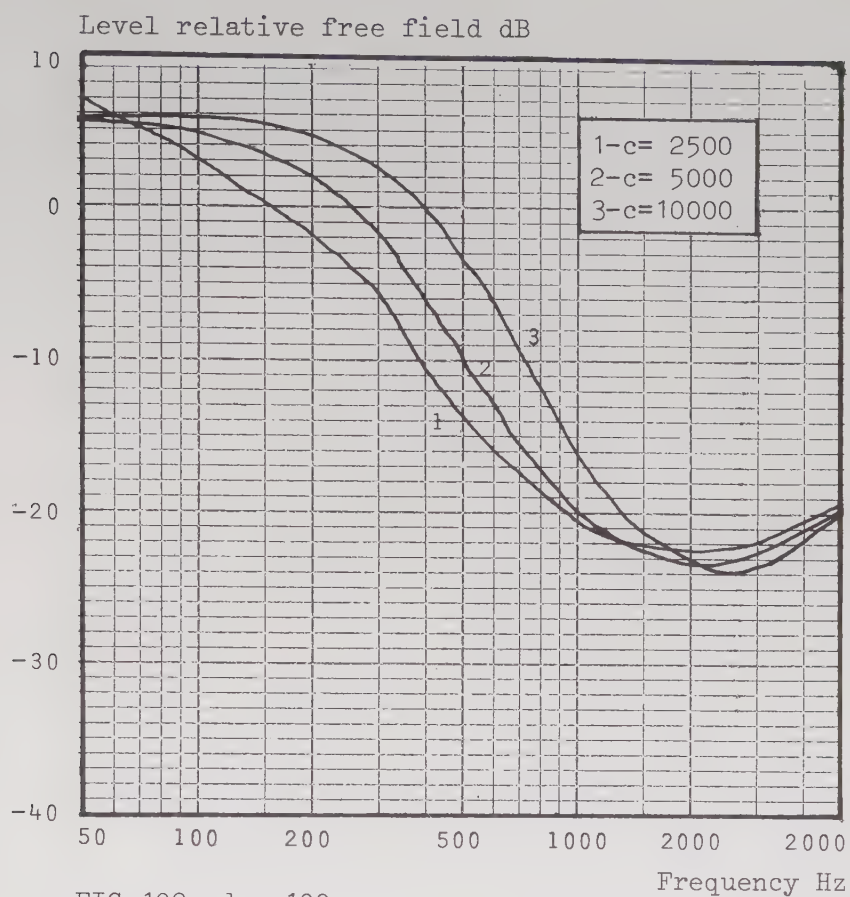


FIG.109: d = 100

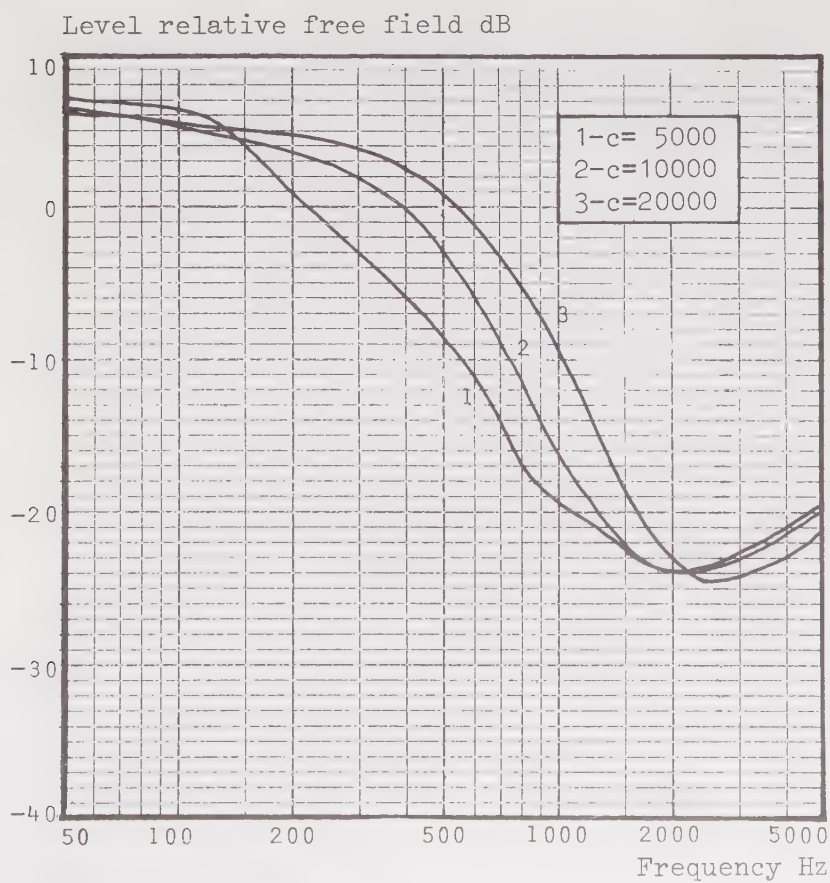


FIG.110: d = 200

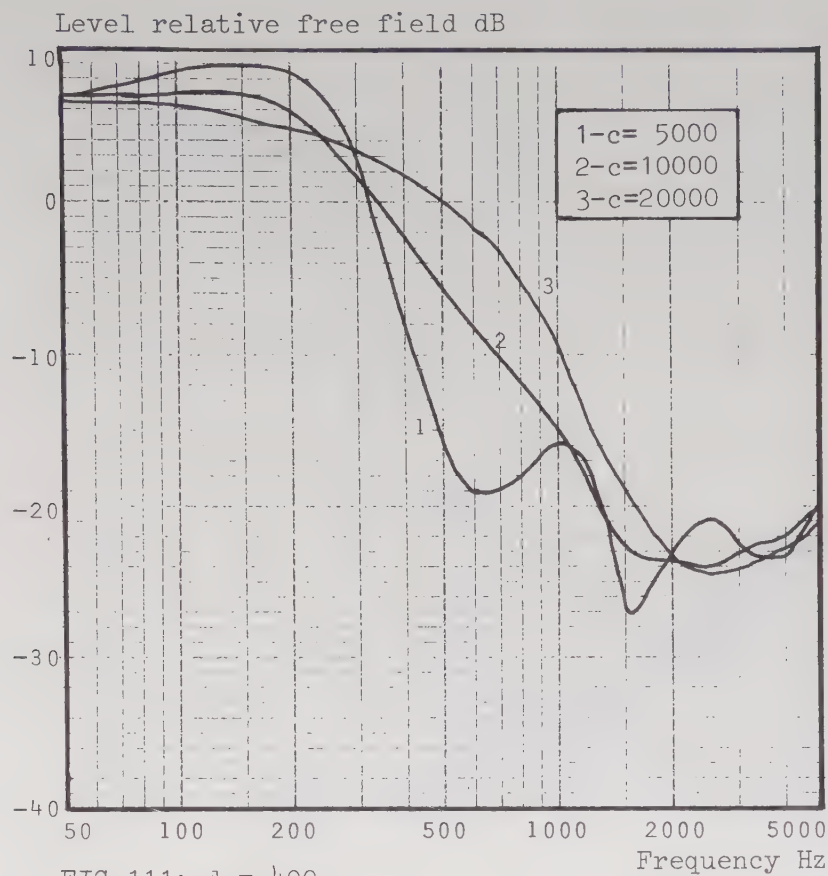


FIG.111: d = 400

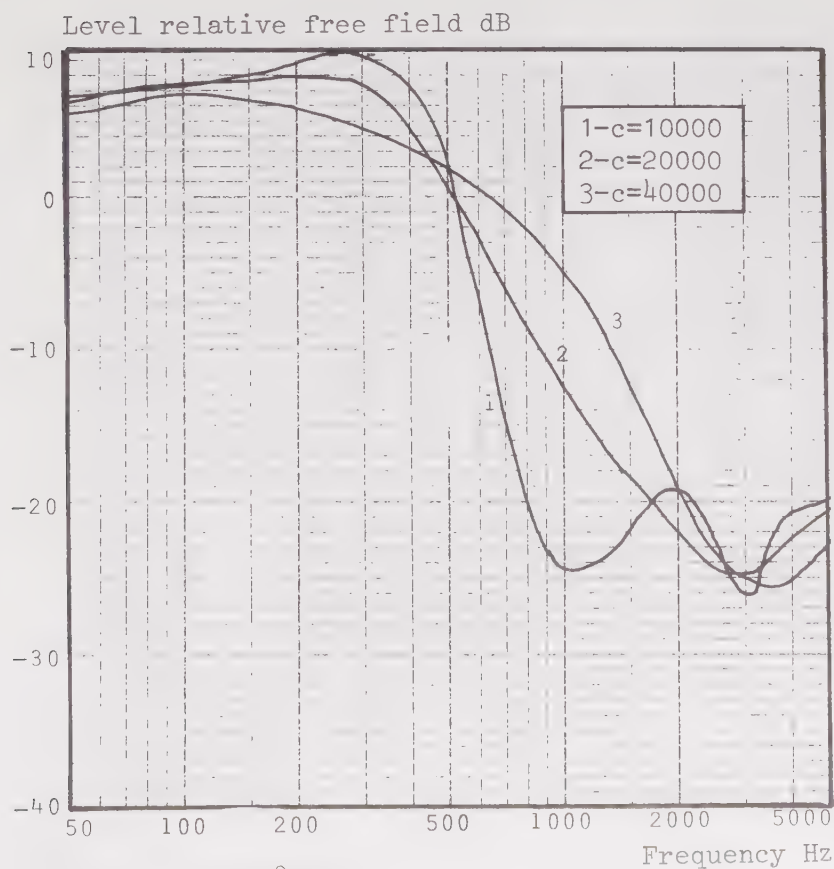
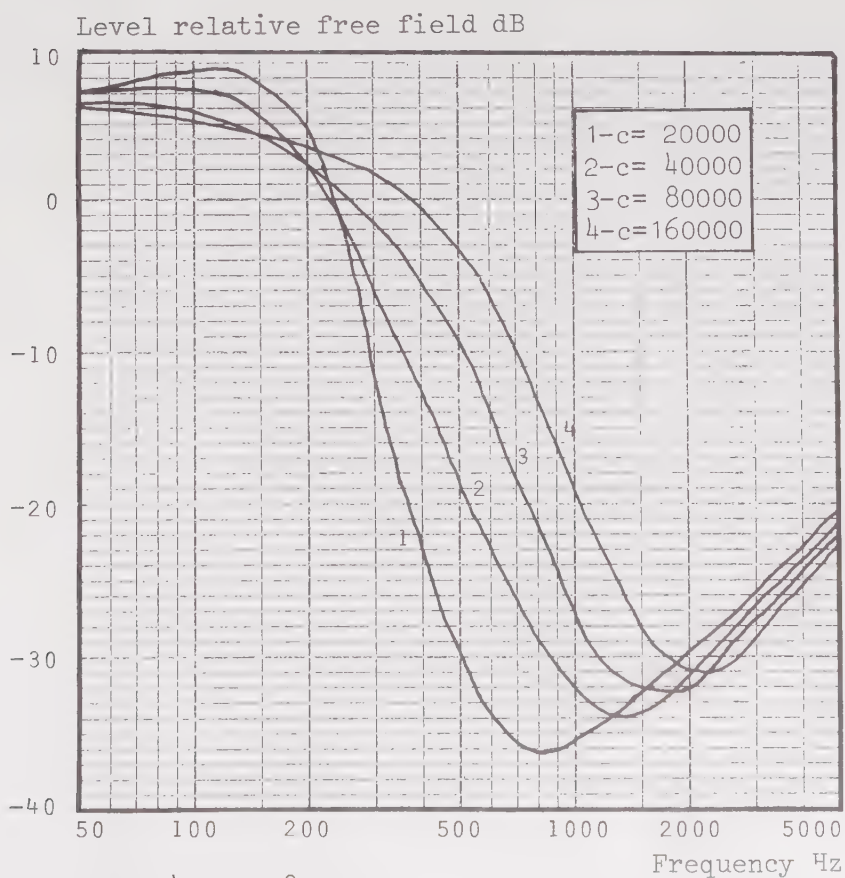
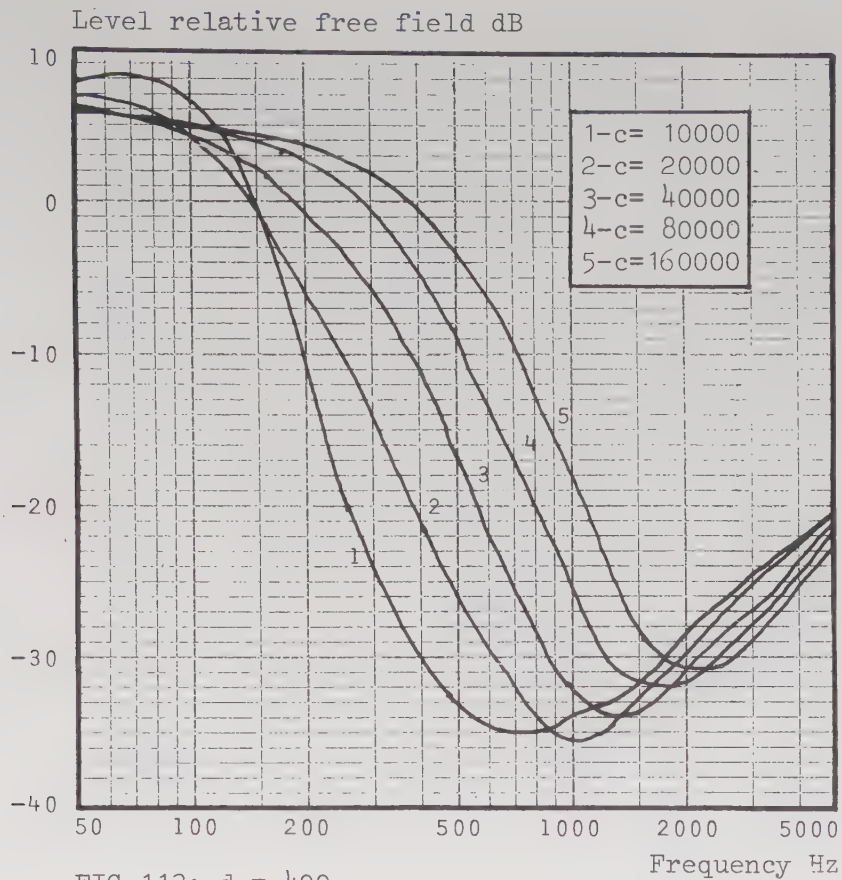


FIG.112: d = 800



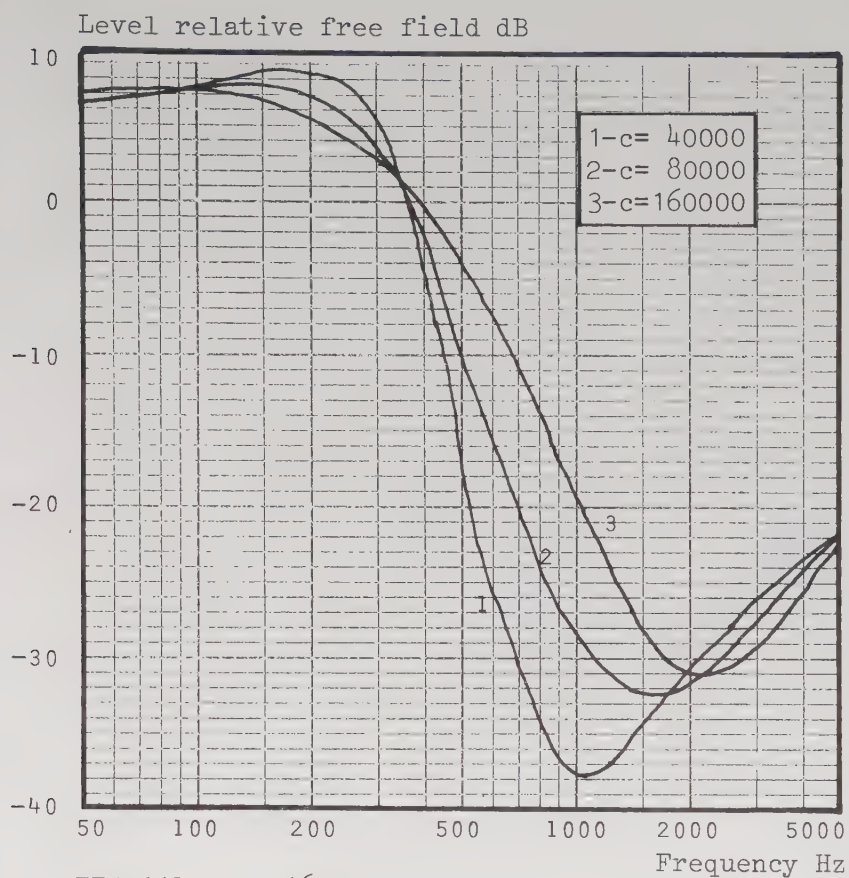


FIG.115: d = 1600

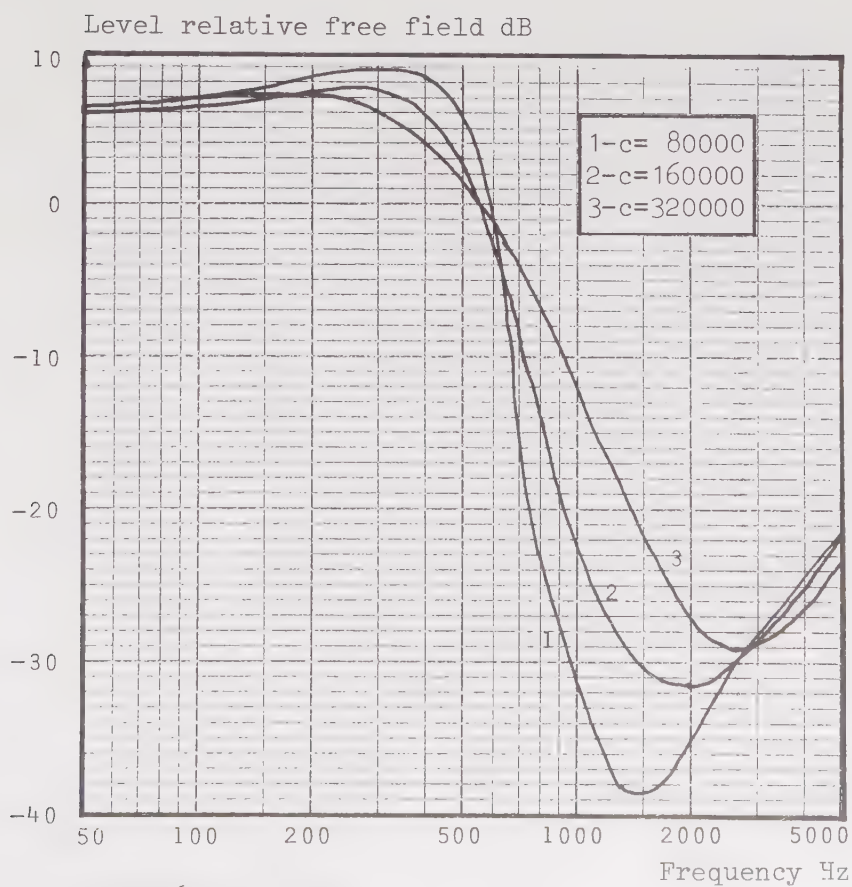


FIG.116: d = 3200

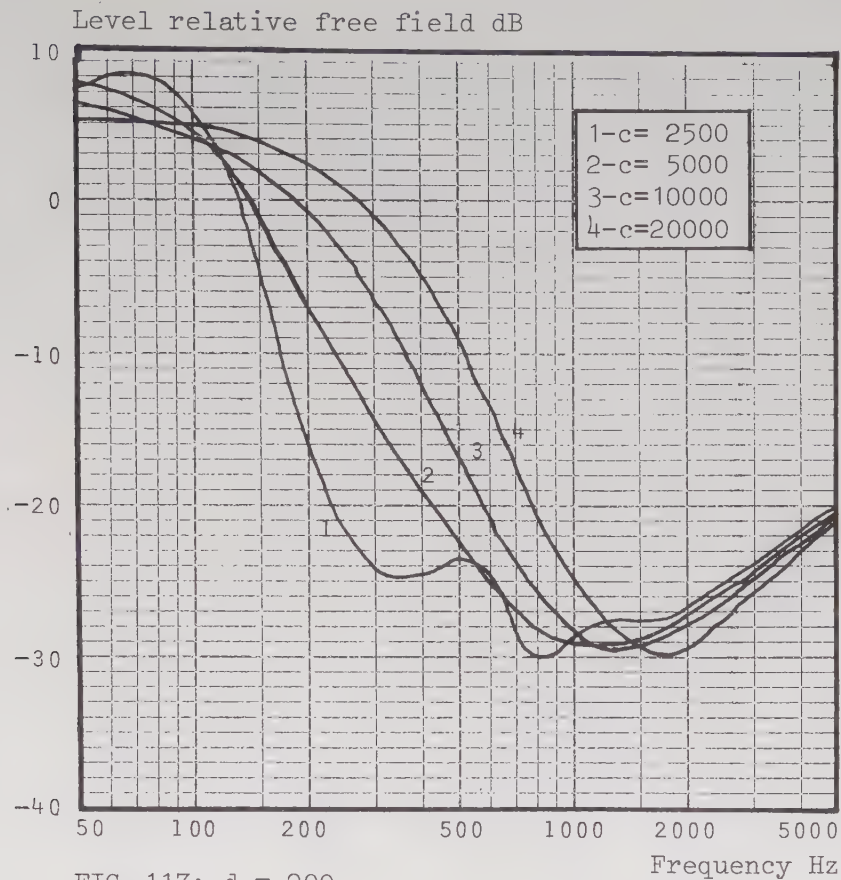


FIG. 117: d = 200

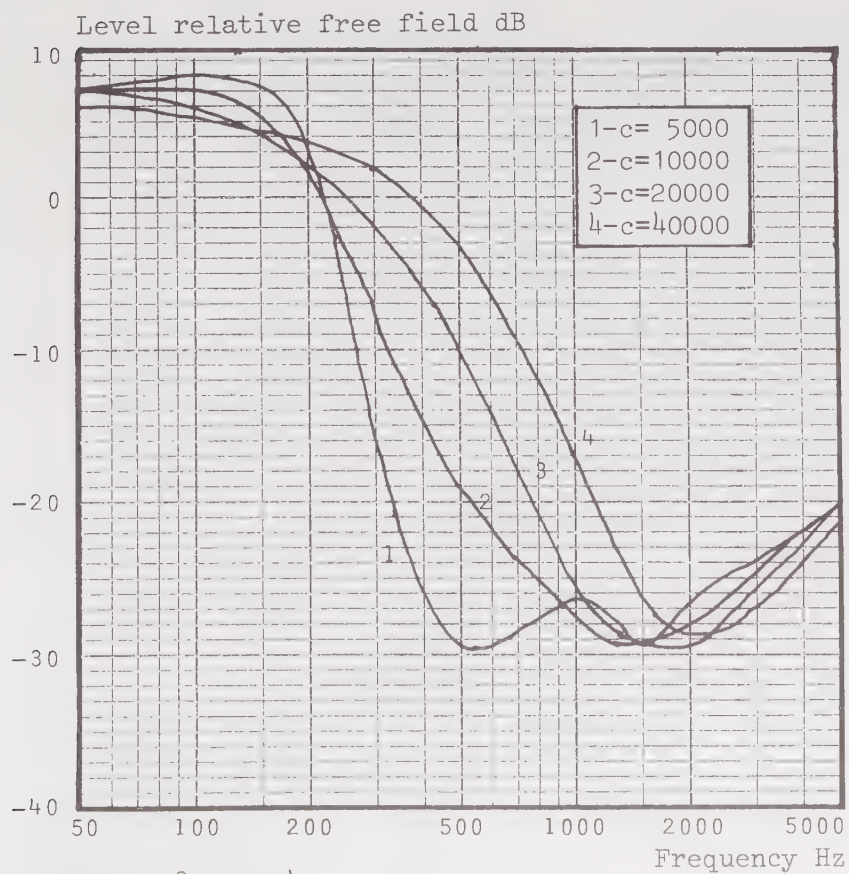
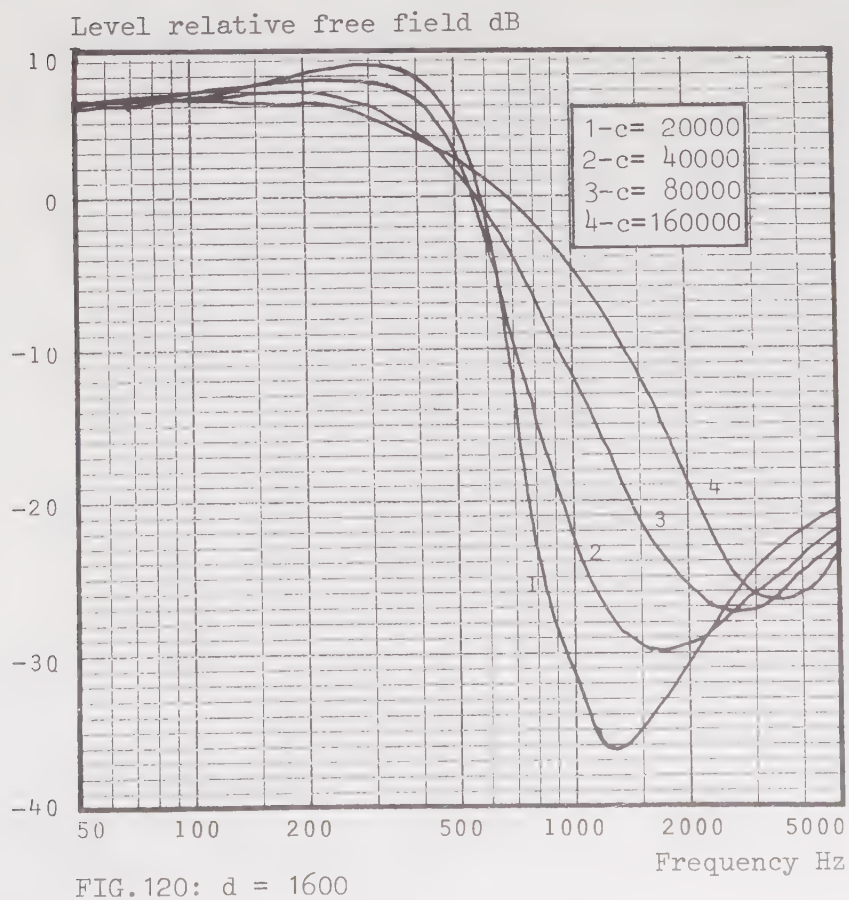
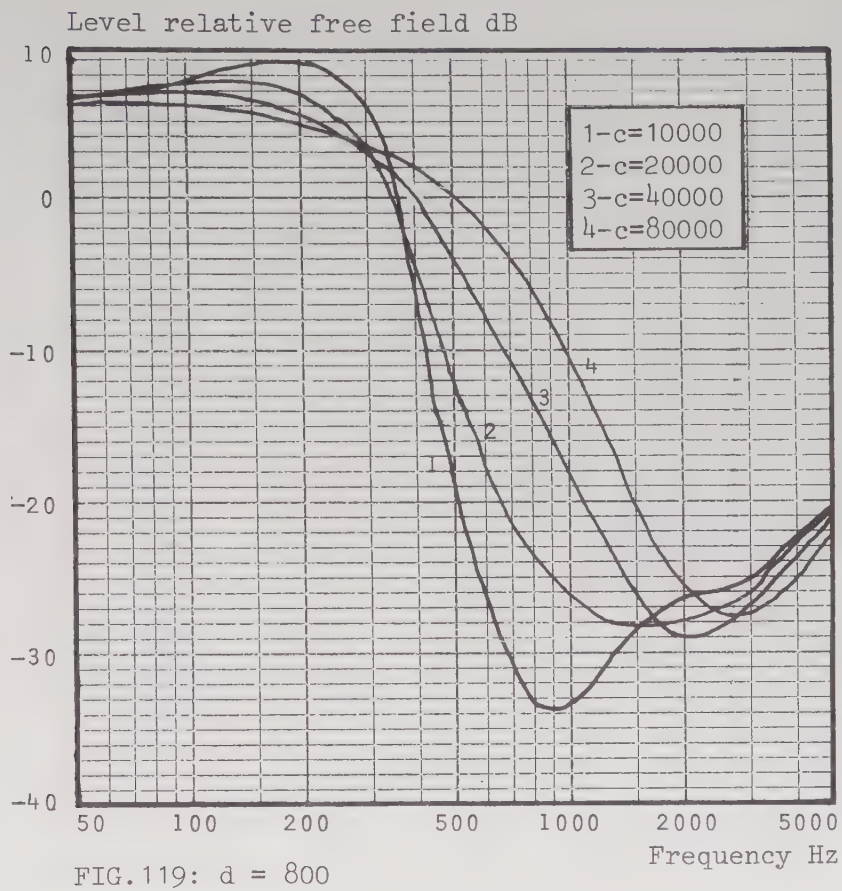


FIG. 118: d = 400



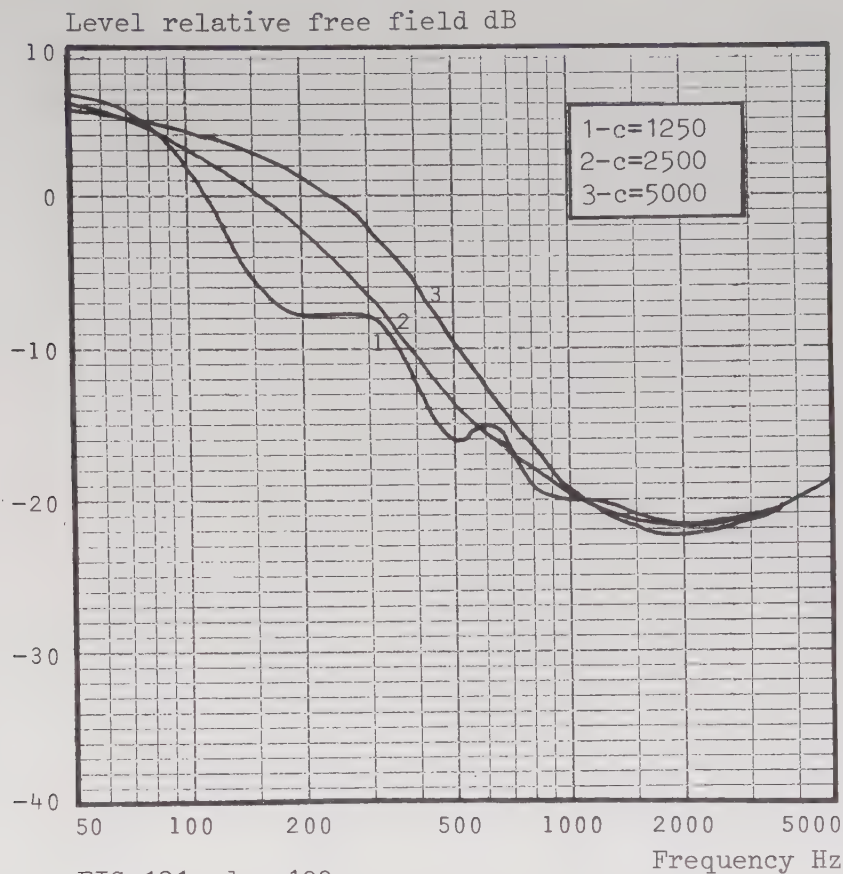


FIG.121: d = 100

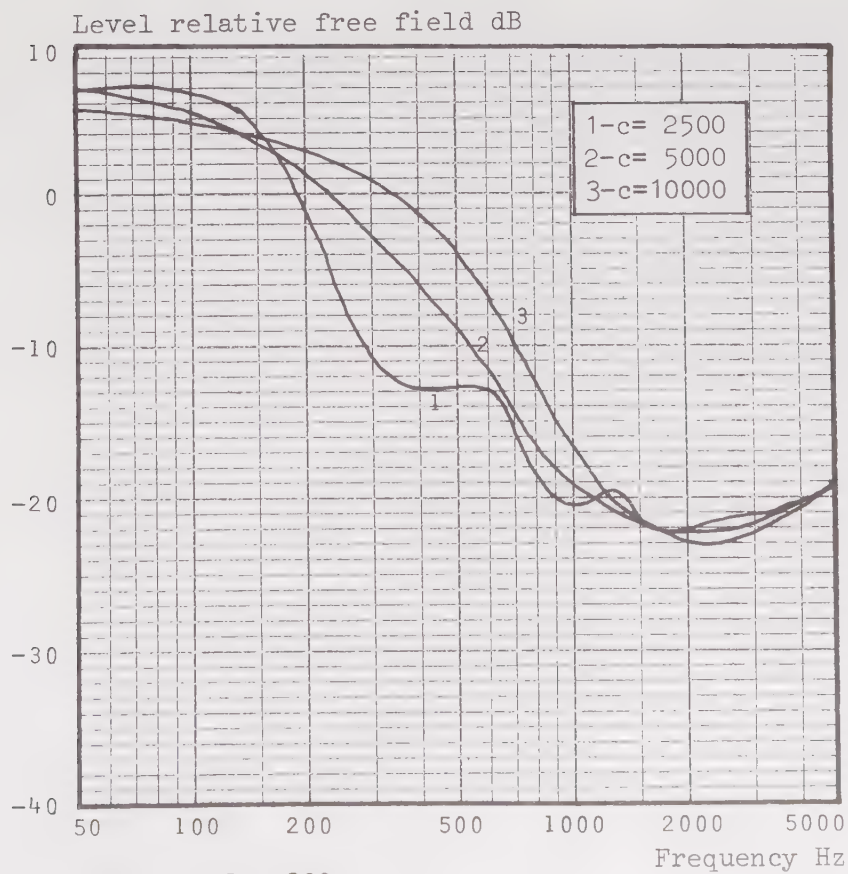
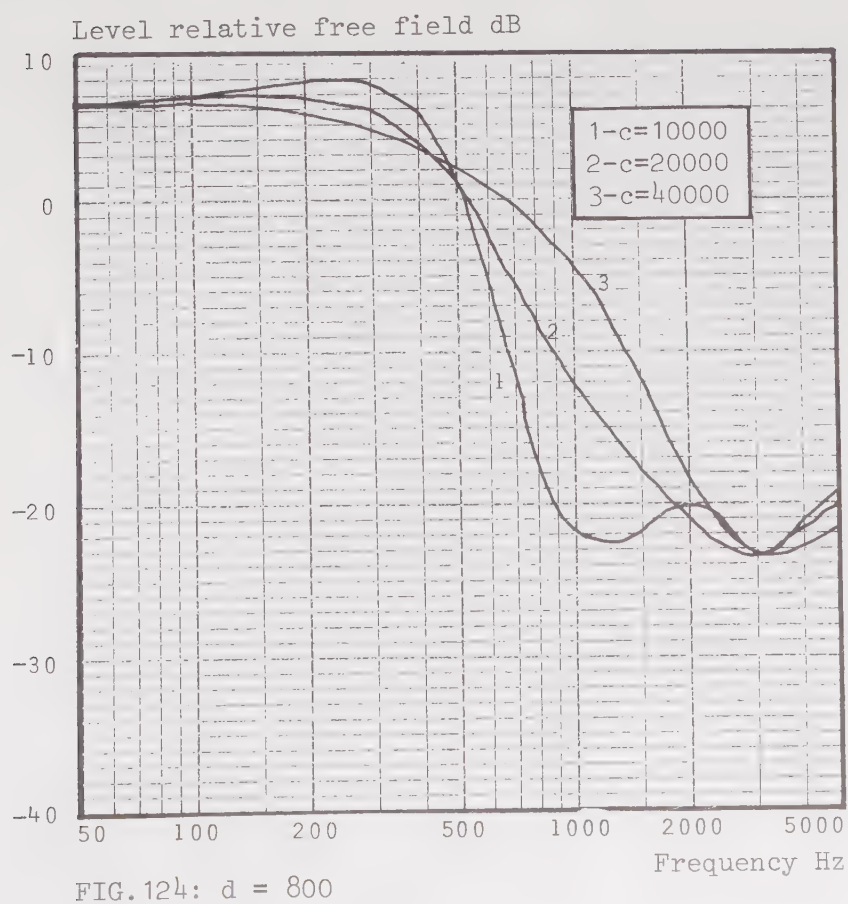
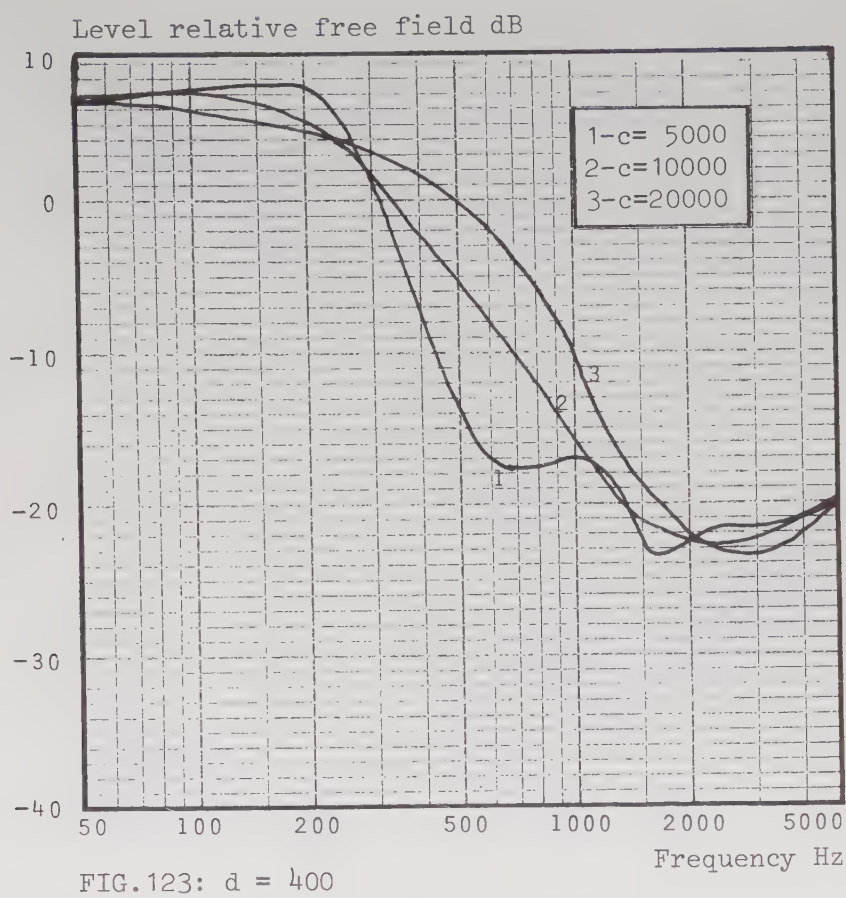


FIG.122: d = 200



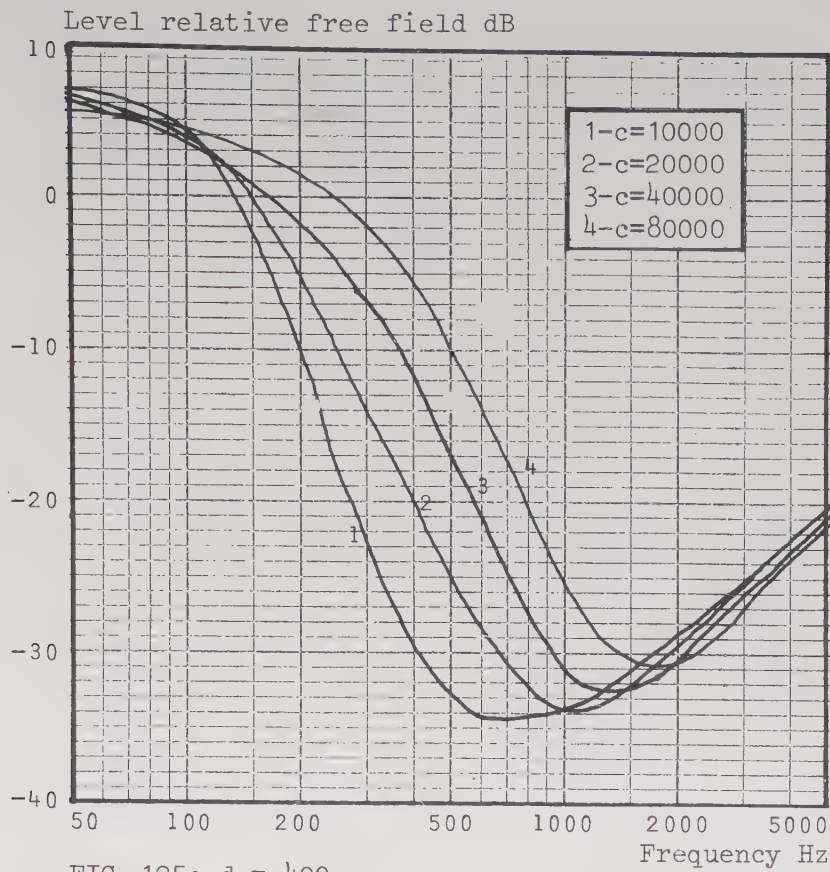


FIG. 125: d = 400

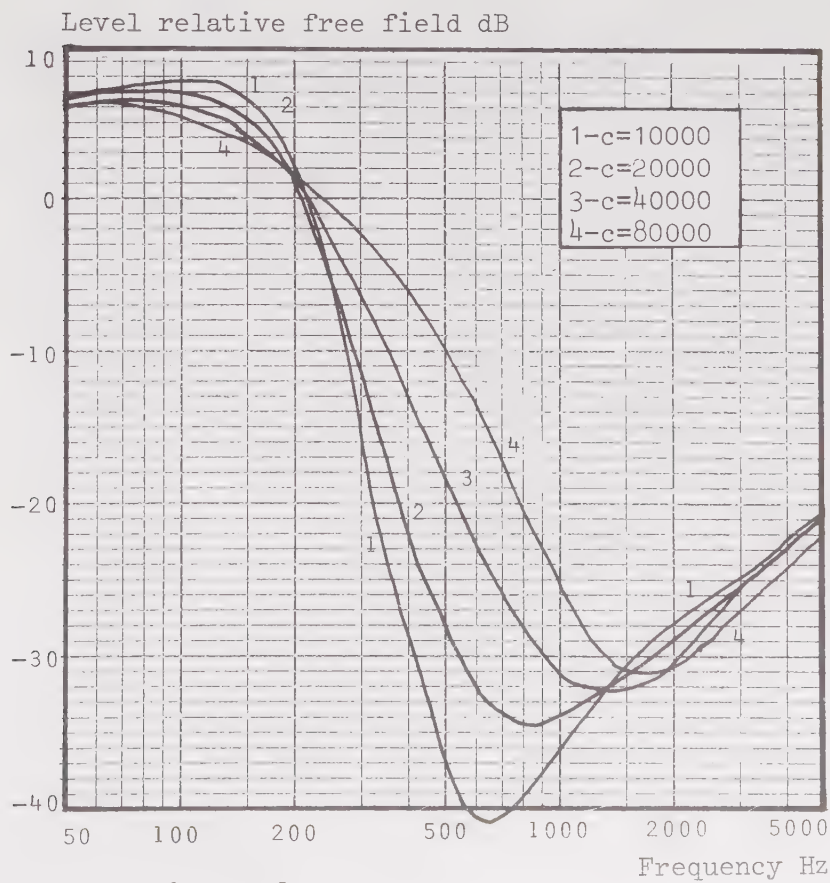


FIG. 126: d = 800

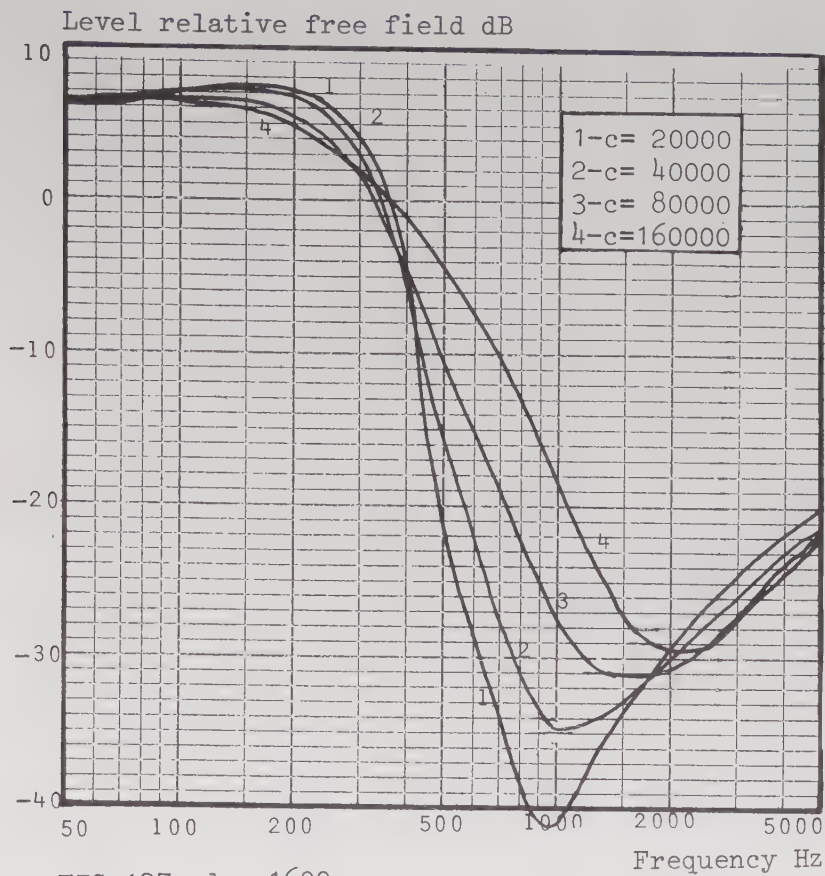


FIG.127: d = 1600

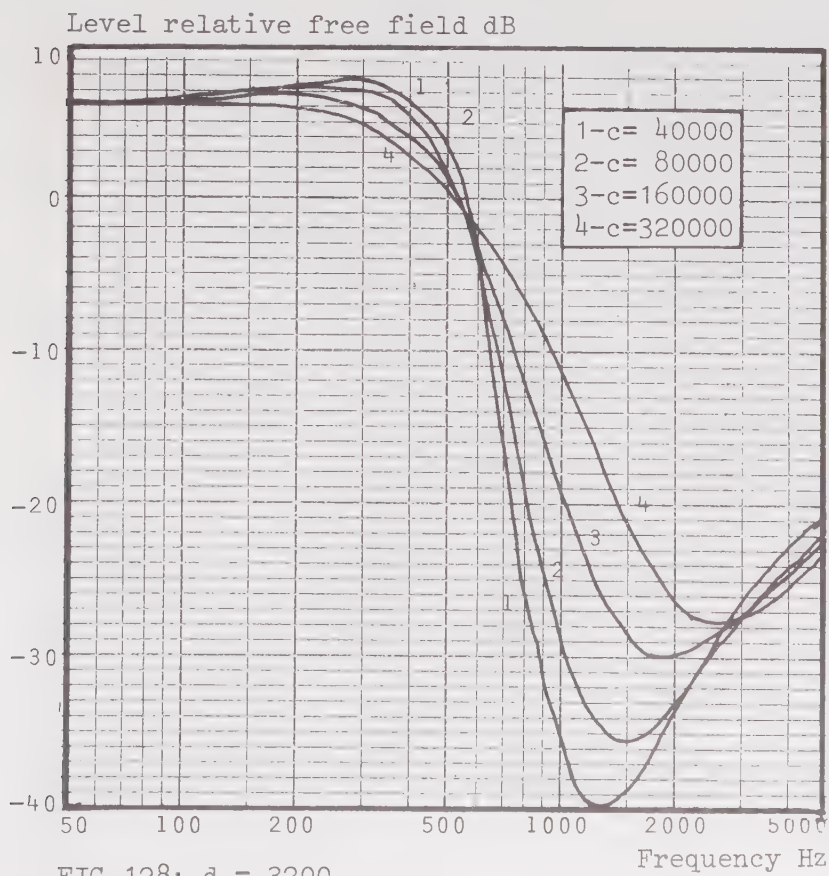
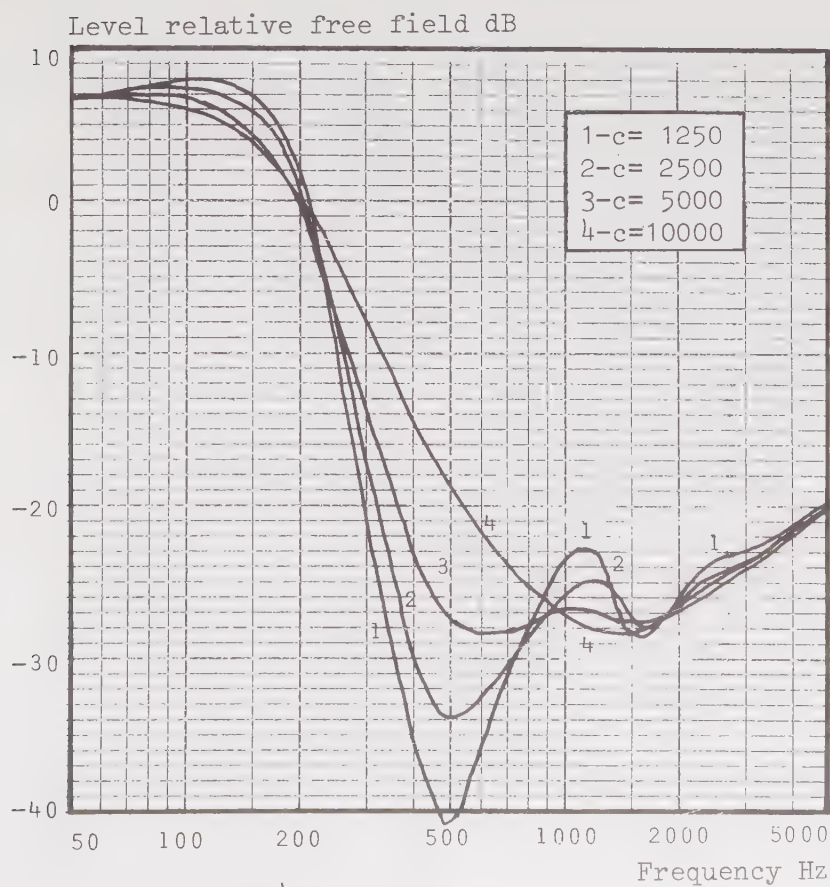
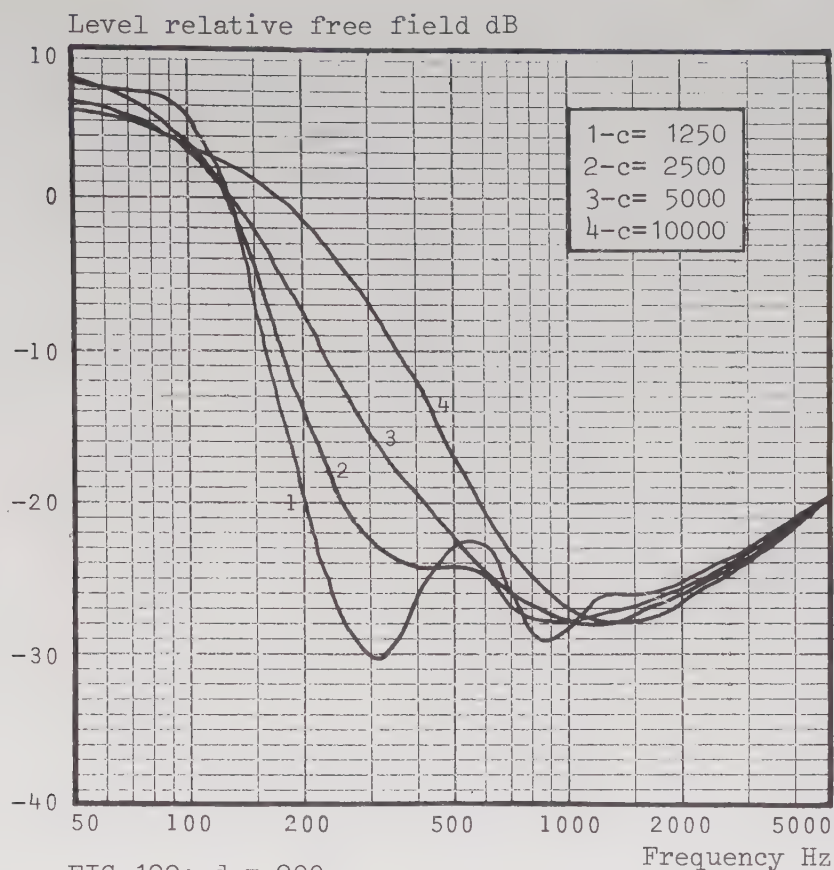
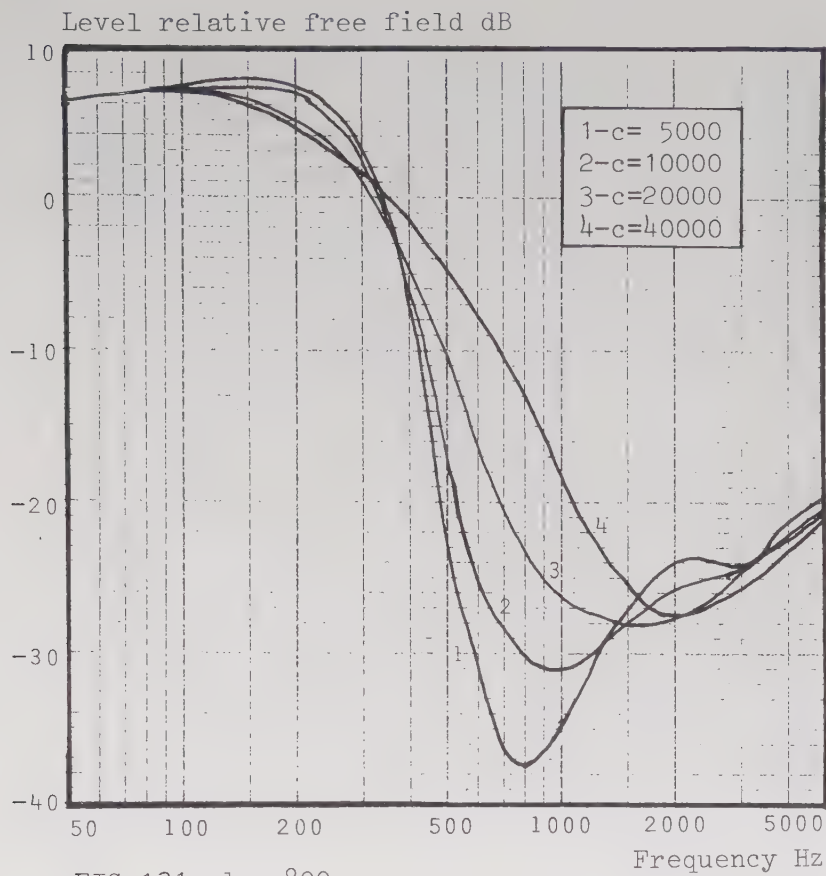
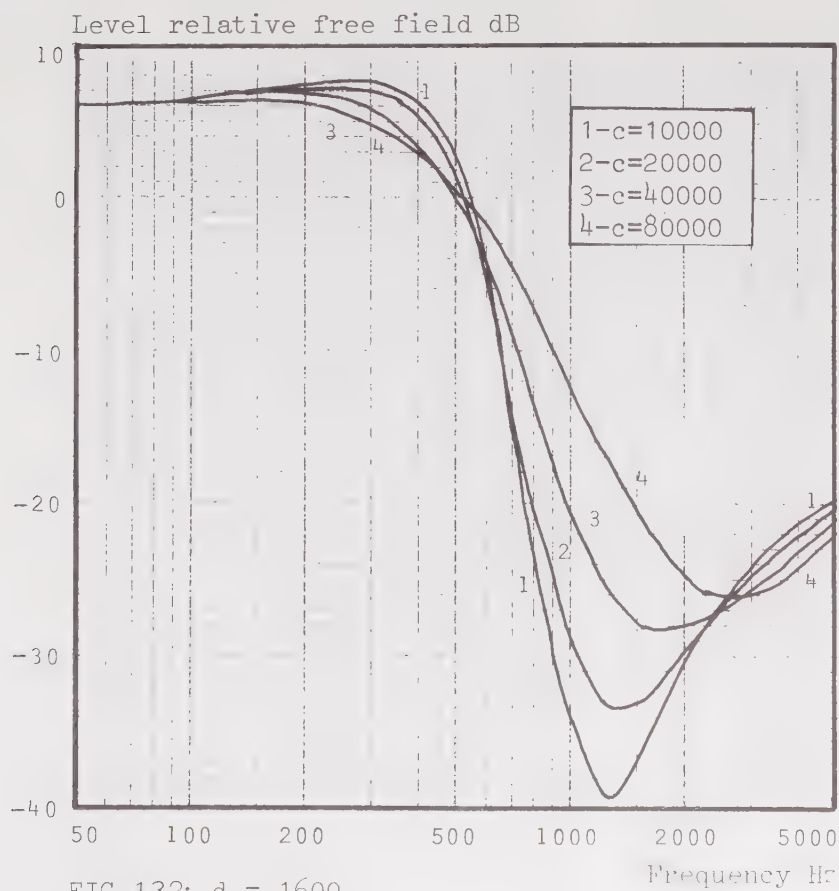


FIG.128: d = 3200




FIG.131: $d = 800$

FIG.132: $d = 1600$

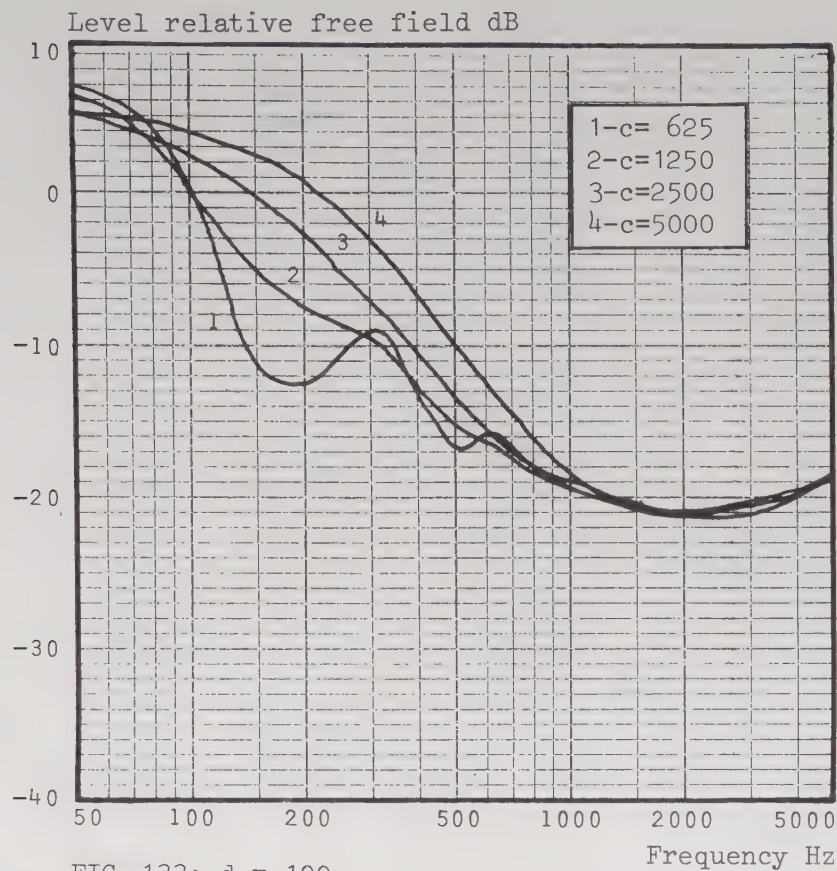


FIG. 133: d = 100

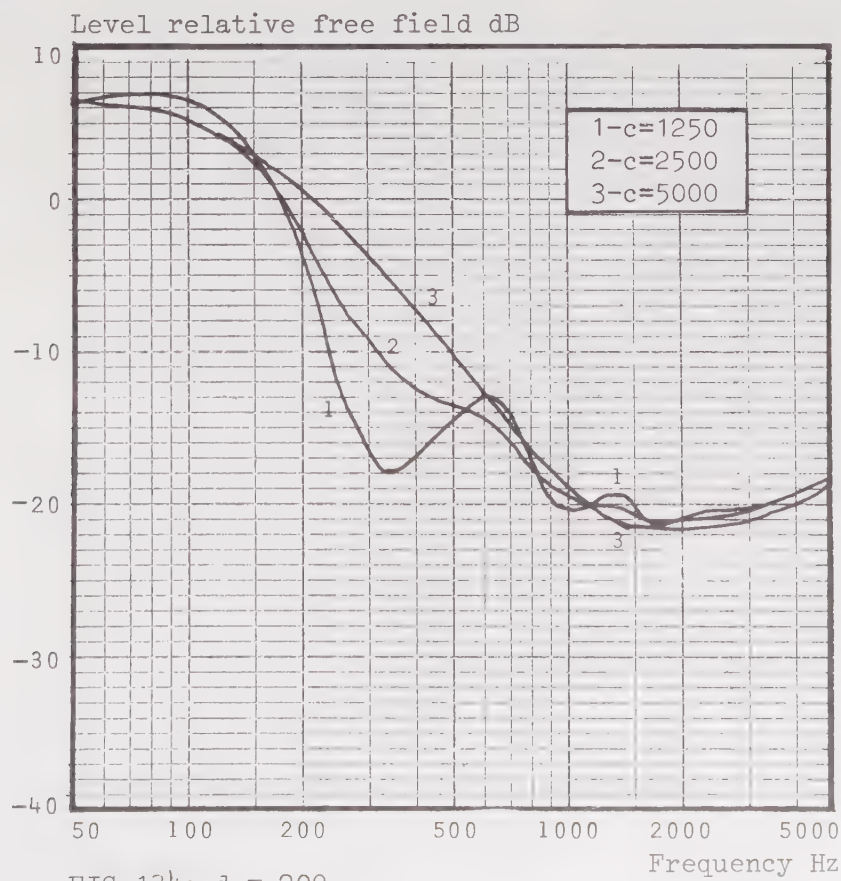


FIG. 134: d = 200

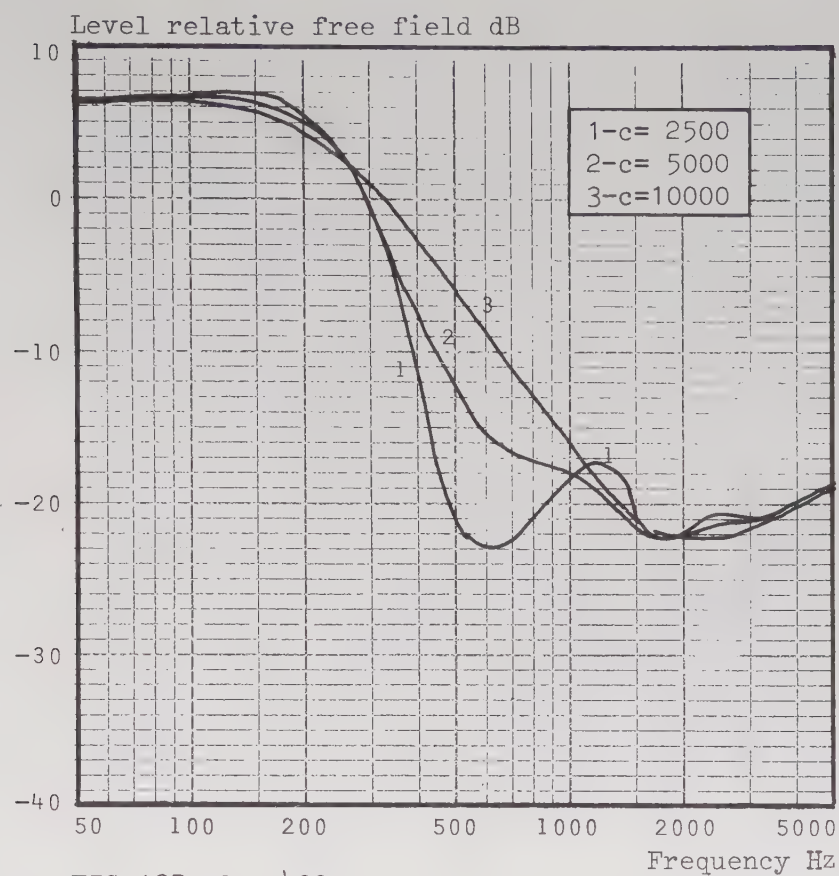


FIG.135: d = 400

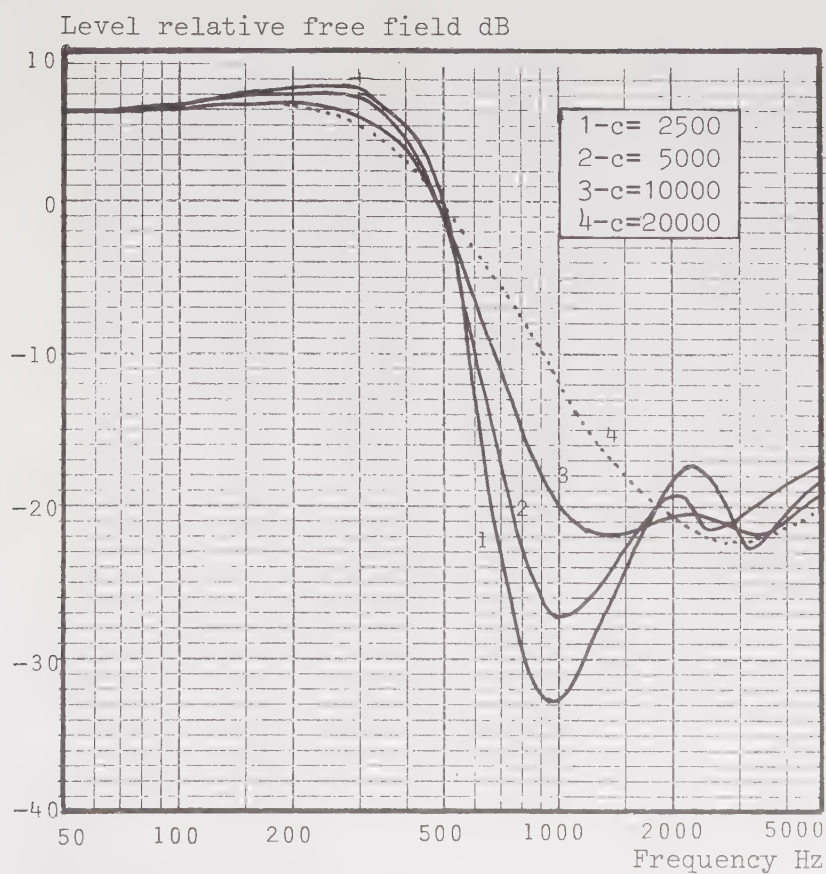


FIG.136: d = 800

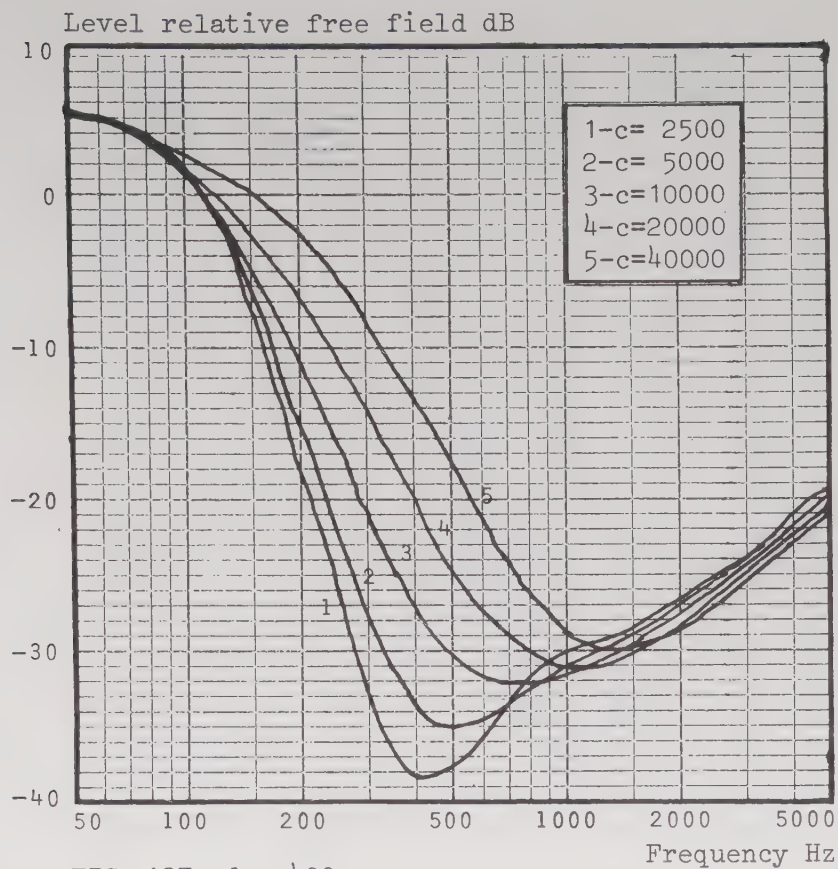


FIG. 137: d = 400

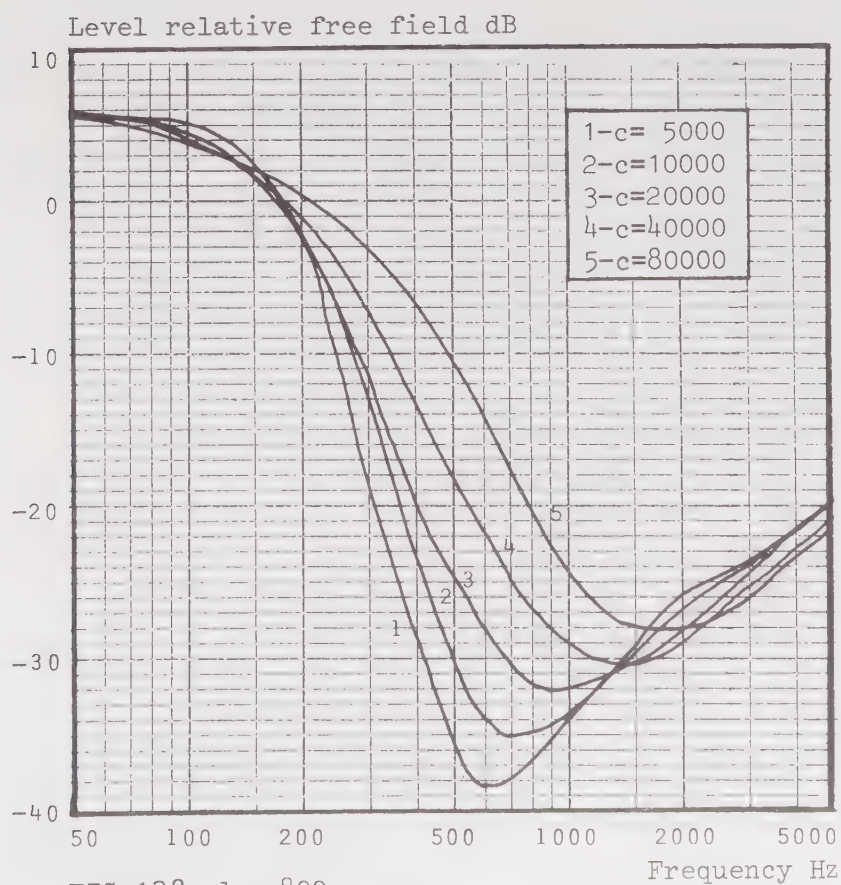


FIG. 138: d = 800

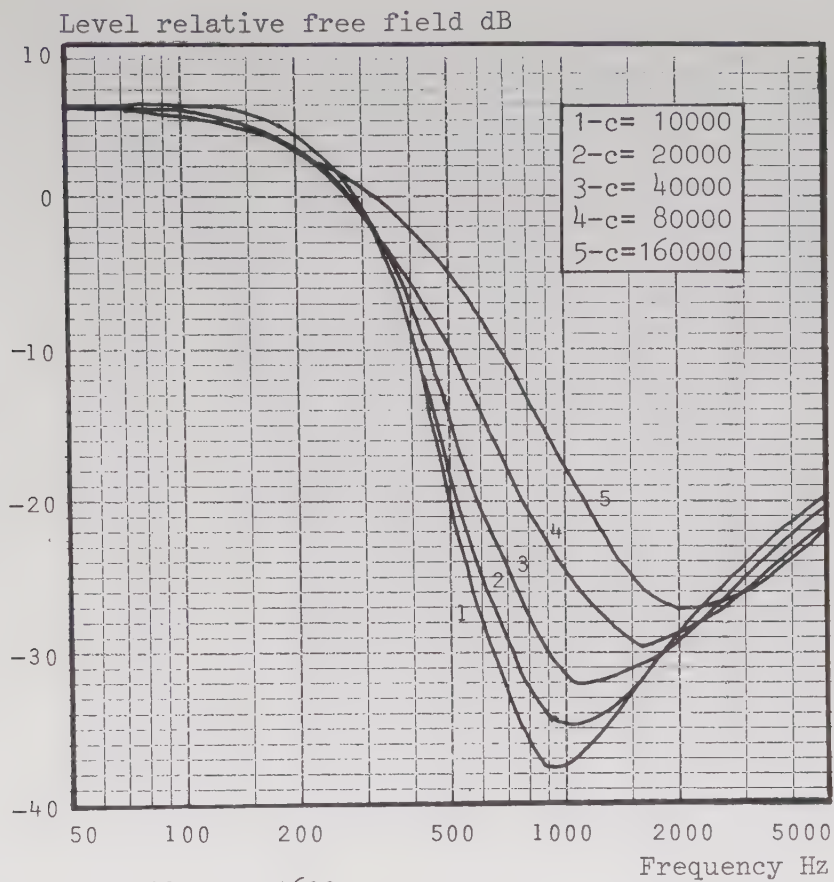


FIG.139: d = 1600

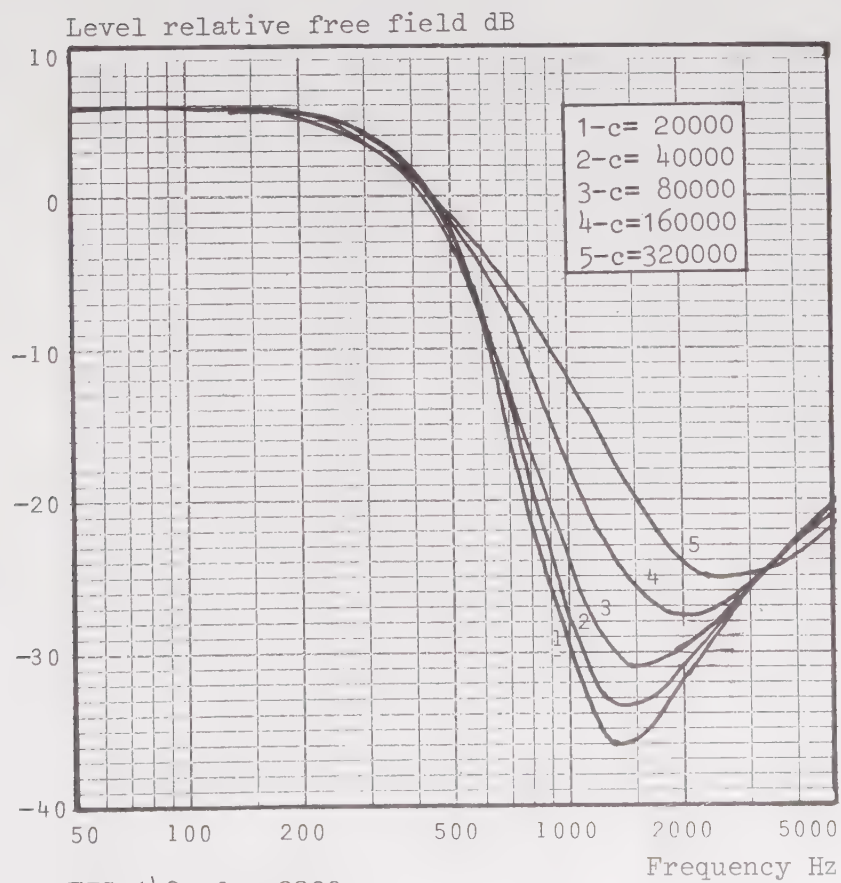


FIG.140: d = 3200

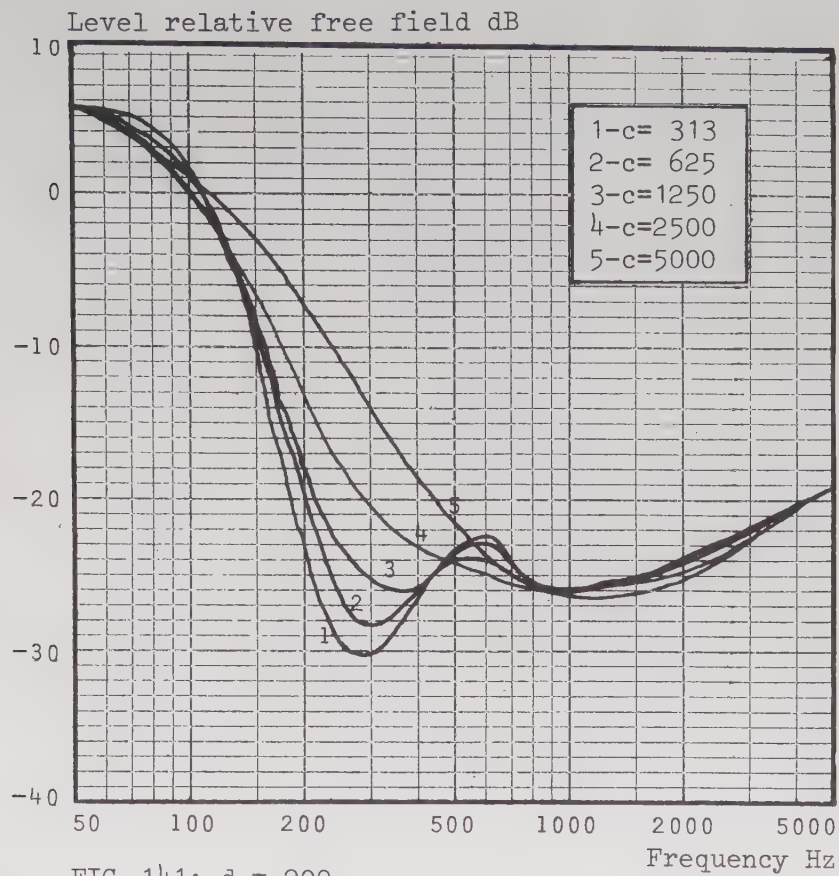


FIG. 141: d = 200

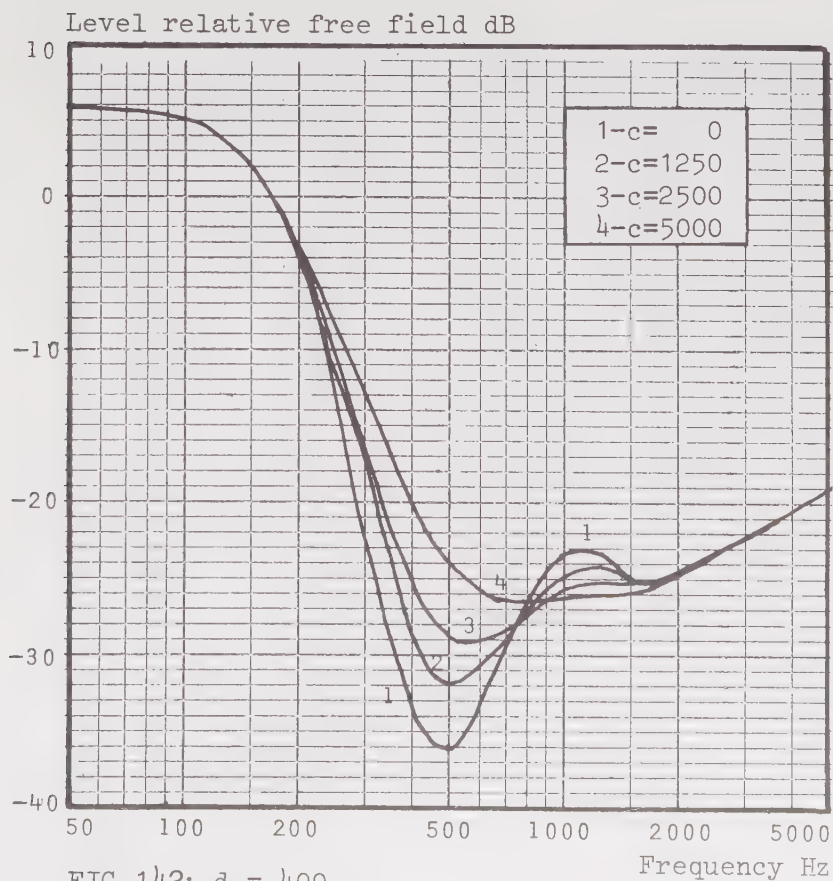
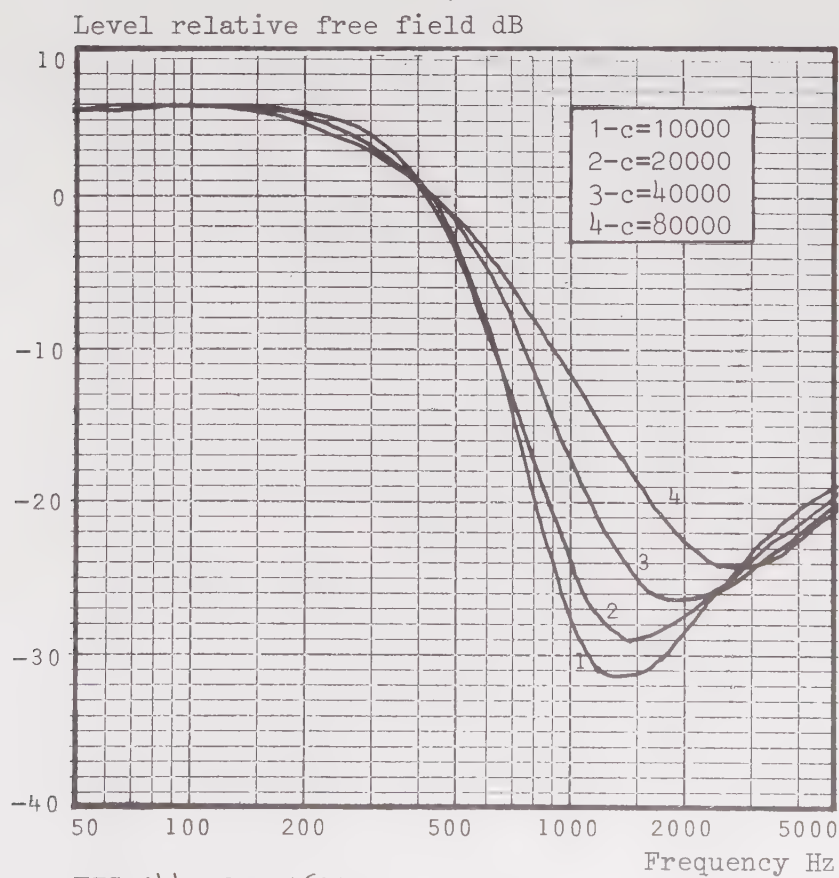
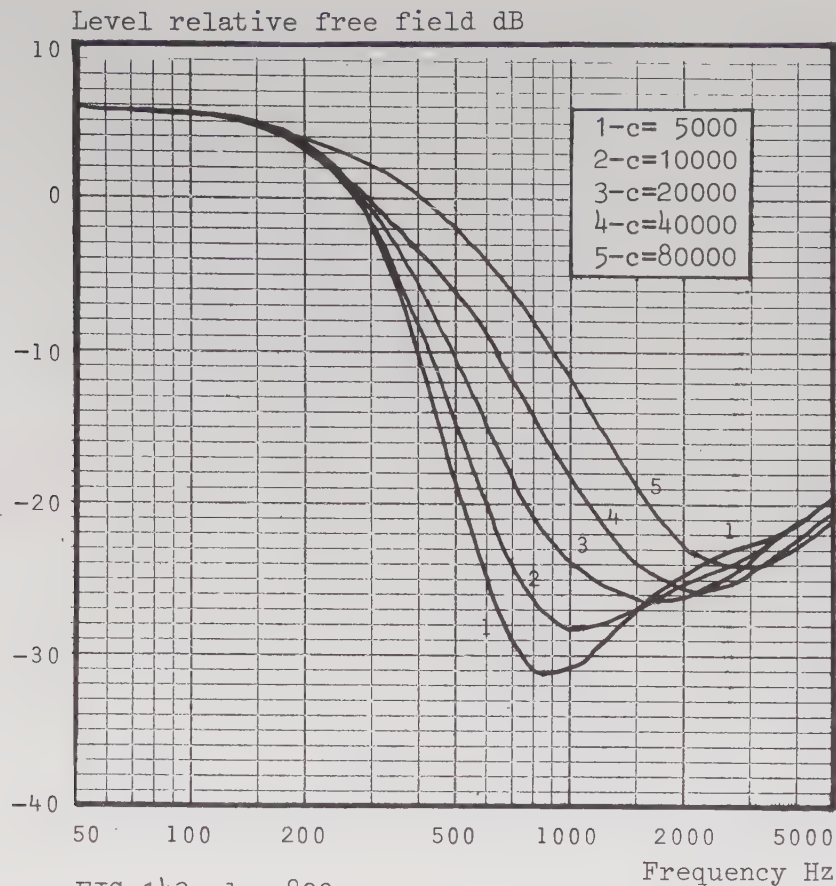


FIG. 142: d = 400



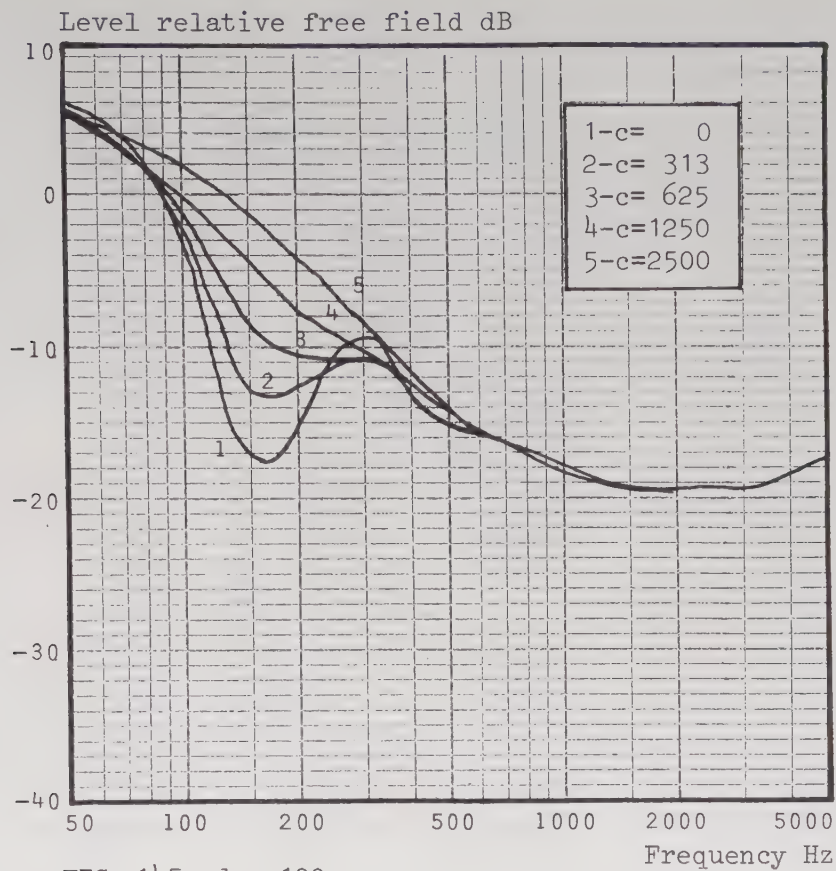


FIG. 145: d = 100

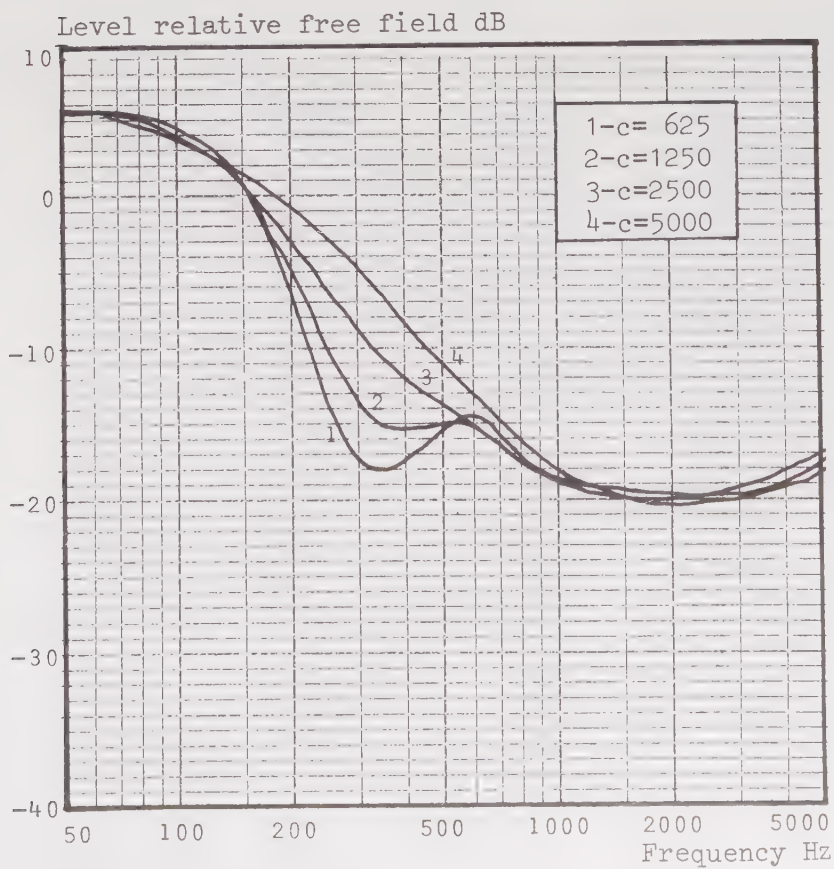


FIG. 146: d = 200

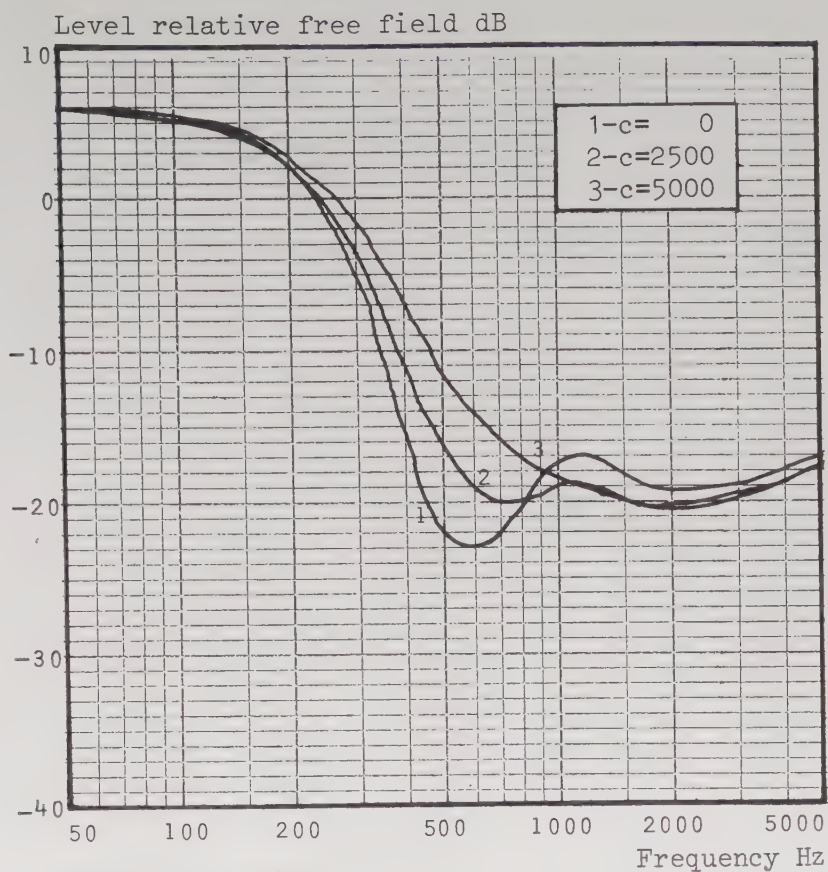


FIG.147: d = 400

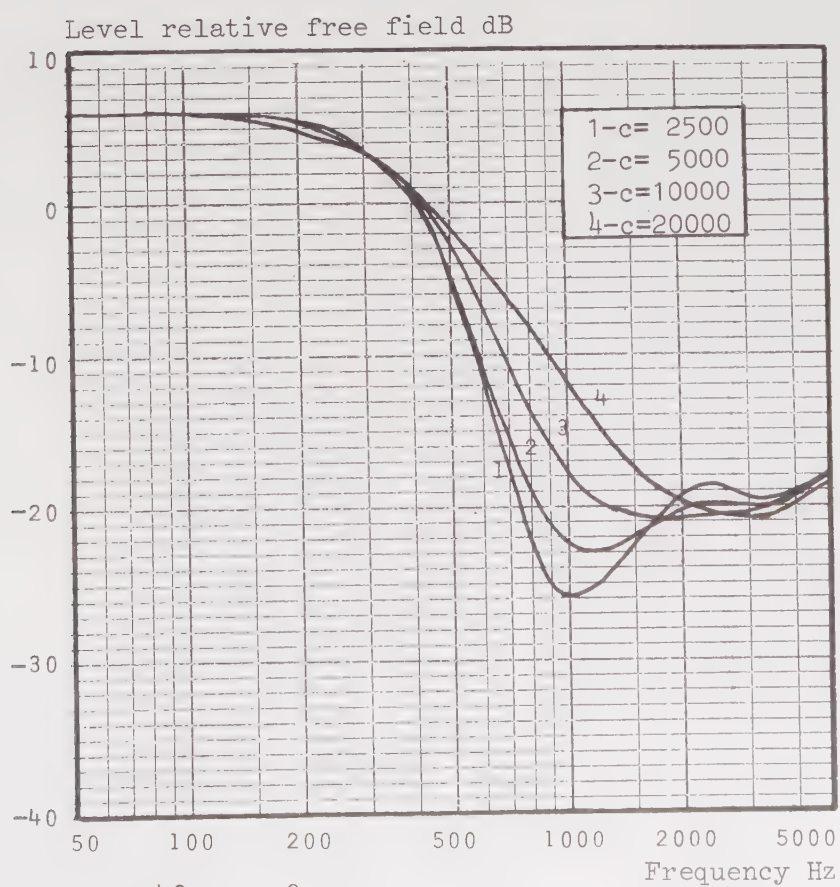


FIG.148: d = 800

S E C T I O N . 2

SCALE MODEL EXPERIMENTS

The following figures, 201-222, have been obtained from a study in an anechoic chamber.

Ground impedance: See Eq. (5); $a = 0.51$, $b = 0.0^0$, $c = 1.9 \times 10^5$ Hz,
 $d = 8700$ Hz.

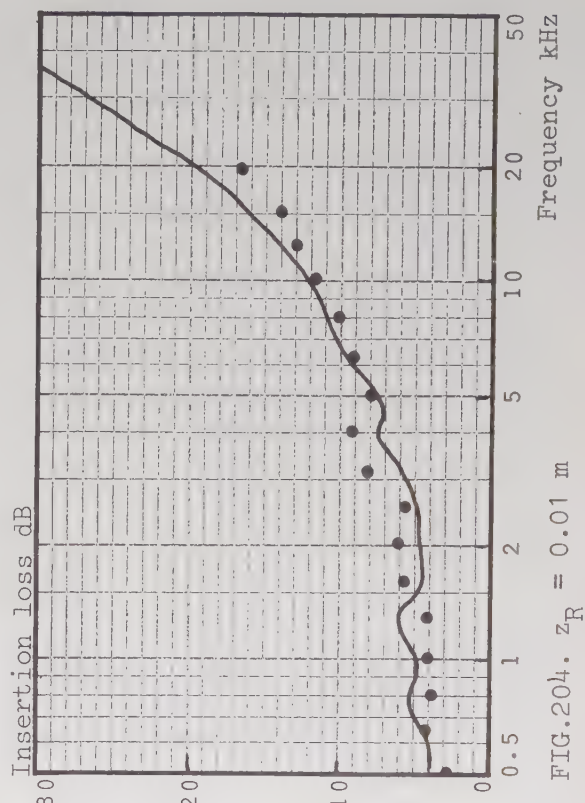
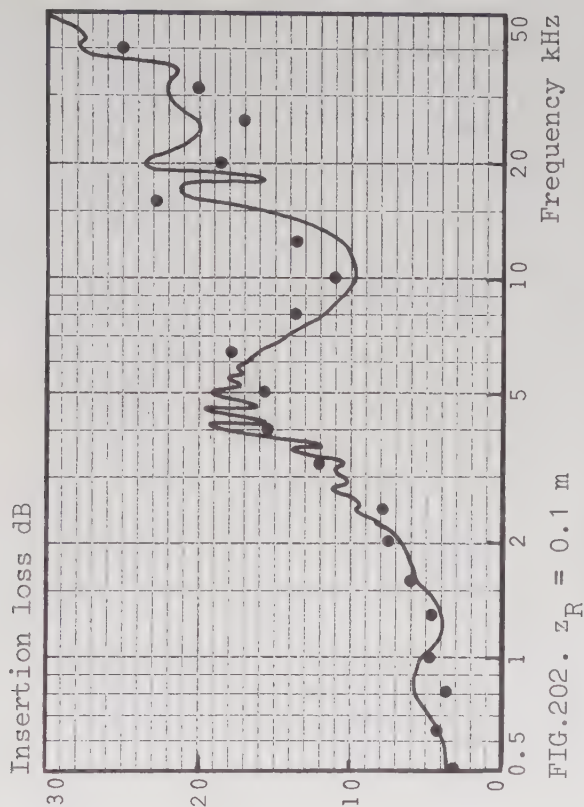
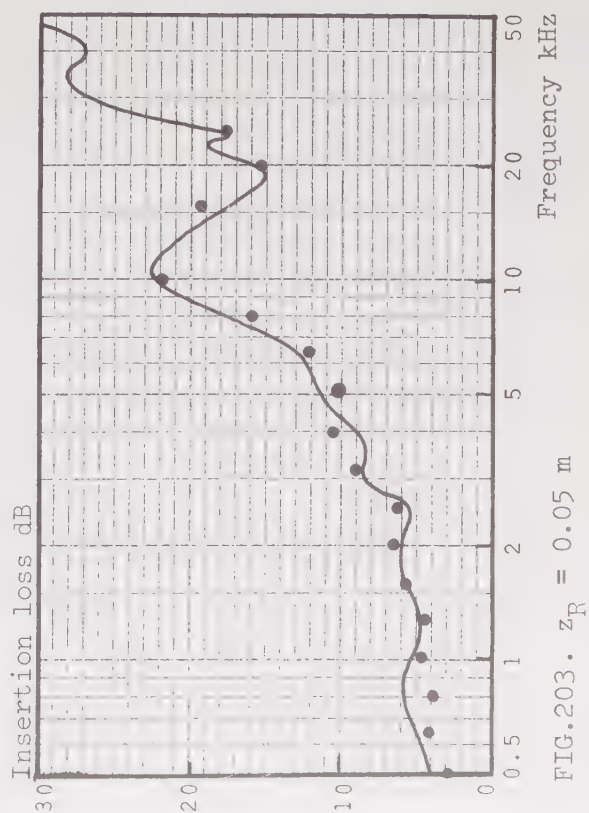
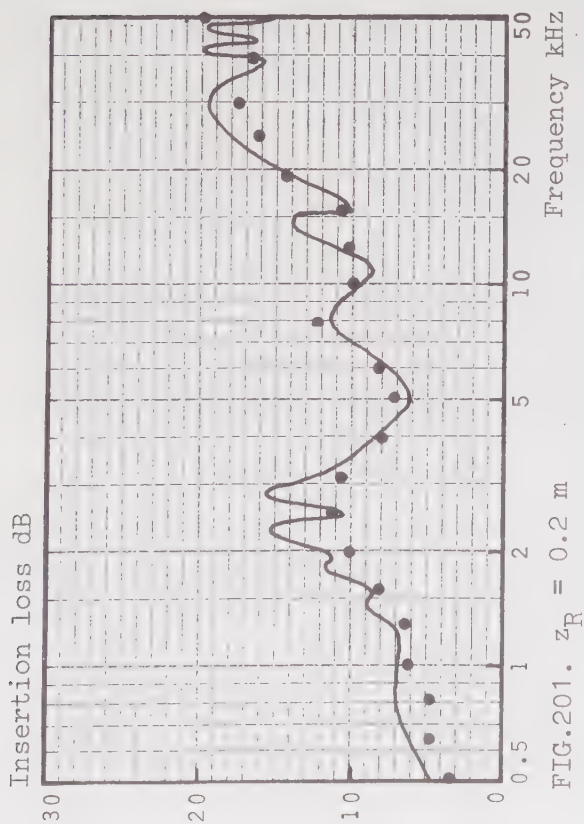
Geometry: See head of each page and compare with Fig. 7.

Calculations: With Eqs. (5), (8)-(15), (25), (28)-(33) via sub-routine INTEG (Part II).

Notation:
—— measured,
• • • calculated.

$$(y_0, y_1, z_1) = (-1.0, 1.0, 0.15) \text{ m}$$

$$(x_S, y_S, z_S) = (1.0, 0.0, 0.01) \text{ m} \quad (x_R, y_R, z_R) = (-1.0, 0.0, z_R) \text{ m}$$



$$(y_0, y_1, z_1) = (-1.0, 1.0, 0.15) \text{ m}$$

$$(x_S, y_S, z_S) = (1.0, 0.0, 0.01) \text{ m} \quad (x_R, y_R, z_R) = (-1.0, 0.0, z_R) \text{ m}$$

Insertion loss dB

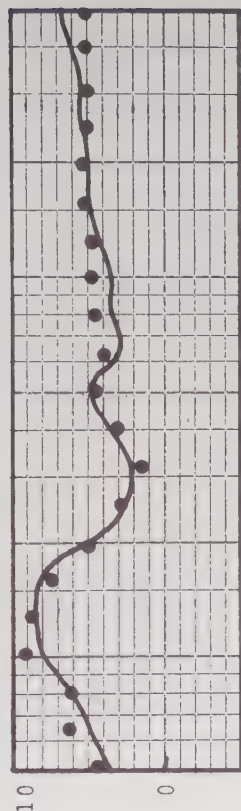


FIG.206. $z_R = 0.3 \text{ m}$

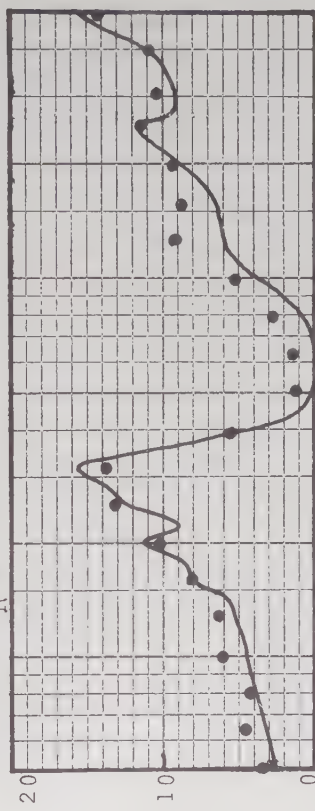


FIG.208. $z_R = 0.1 \text{ m}$

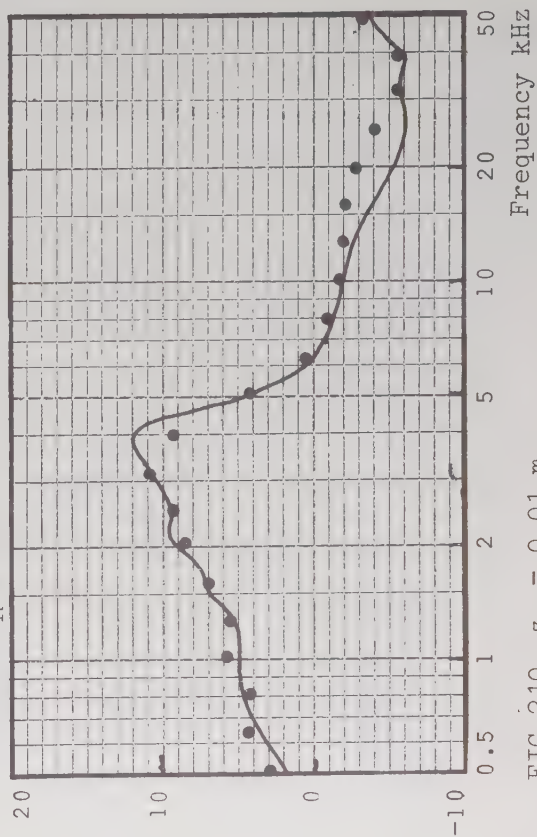


FIG.210. $z_R = 0.01 \text{ m}$

Insertion loss dB



FIG.205. $z_R = 0.4 \text{ m}$

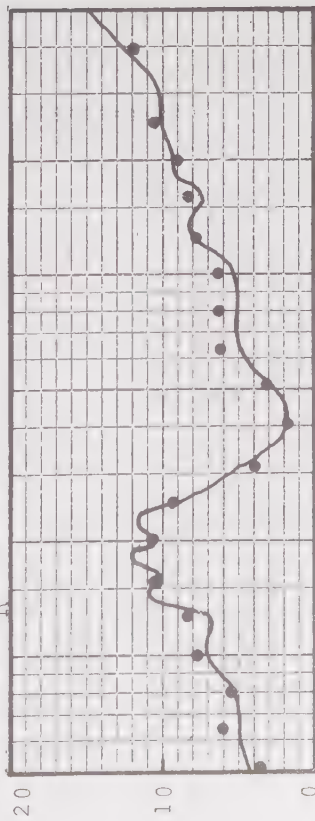


FIG.207. $z_R = 0.2 \text{ m}$

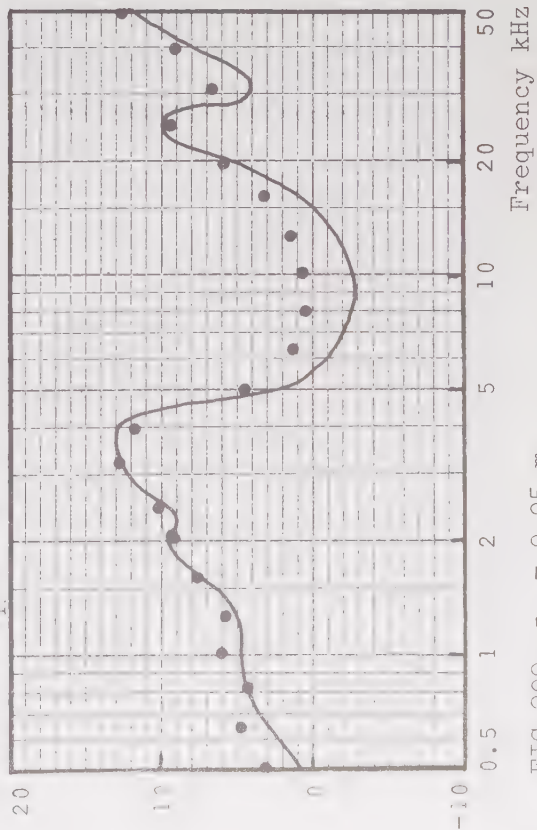


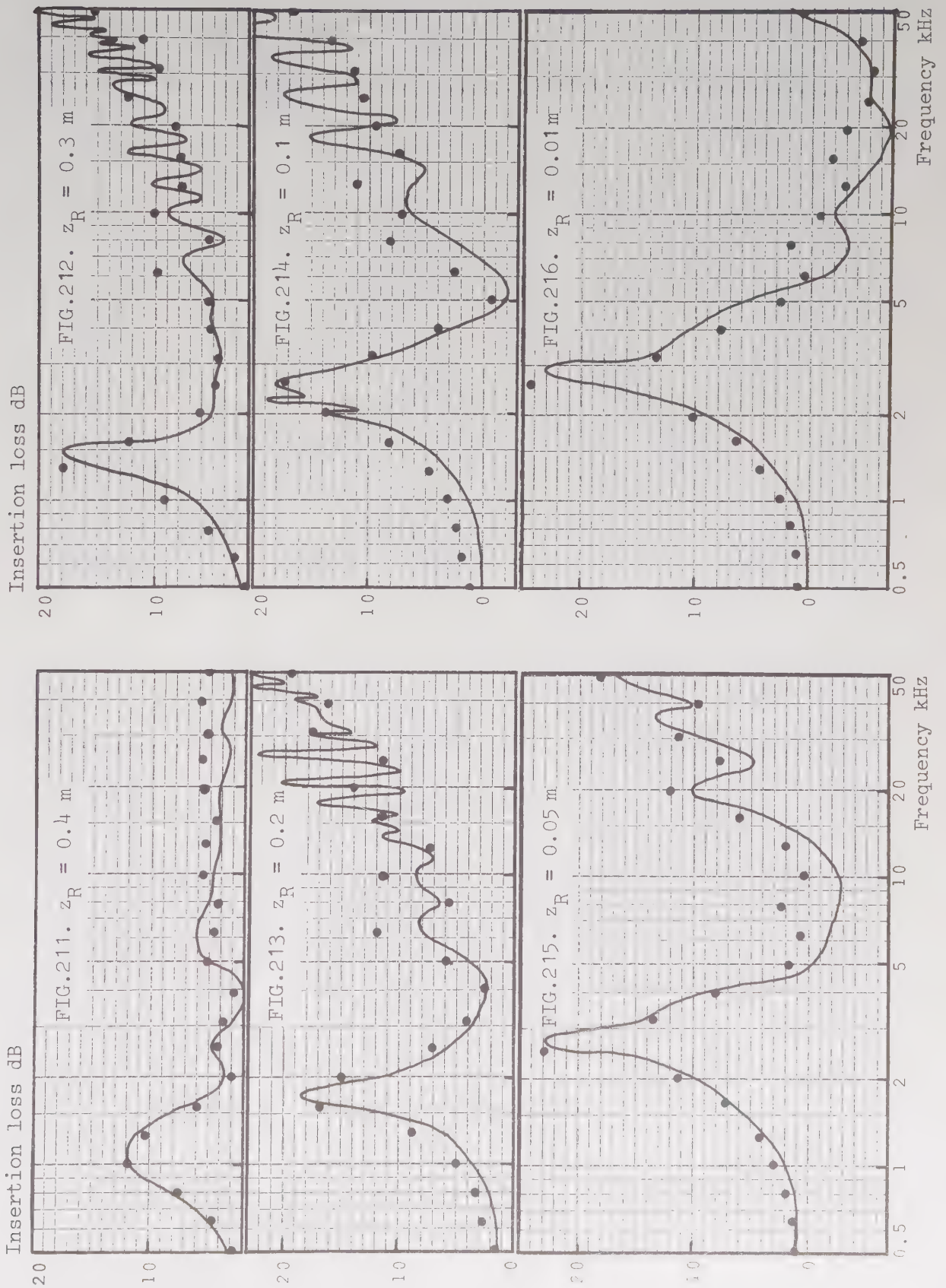
FIG.209. $z_R = 0.05 \text{ m}$

Frequency kHz

Frequency kHz

$$(y_0, y_1, z_1) = (-0.3, 0.3, 0.2) \text{ m}$$

$$(x_S, y_S, z_S) = (1.0, 0.0, 0.01) \text{ m} \quad (x_R, y_R, z_R) = (-1.0, 0.0, z_R) \text{ m}$$



$$(y_0, y_1, z_1) = (-1.0, 1.0, 0.3) \text{ m}$$

$$(x_S, y_S, z_S) = (1.0, 0.0, 0.01) \text{ m} \quad (x_R, y_R, z_R) = (-1.0, 0.0, z_R) \text{ m}$$

Insertion loss dB

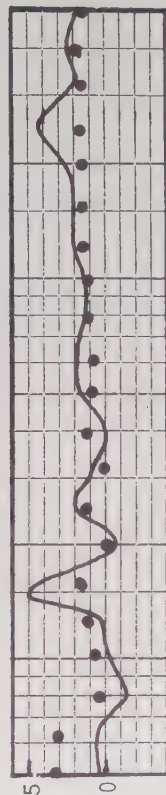


FIG. 217. $z_R = 0.8 \text{ m}$

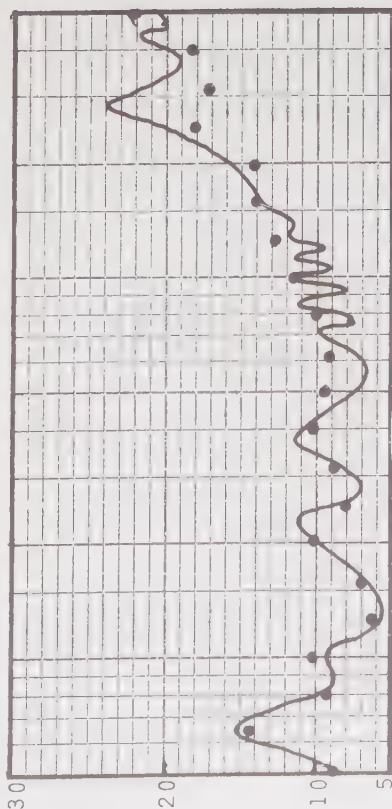


FIG. 219. $z_R = 0.4 \text{ m}$

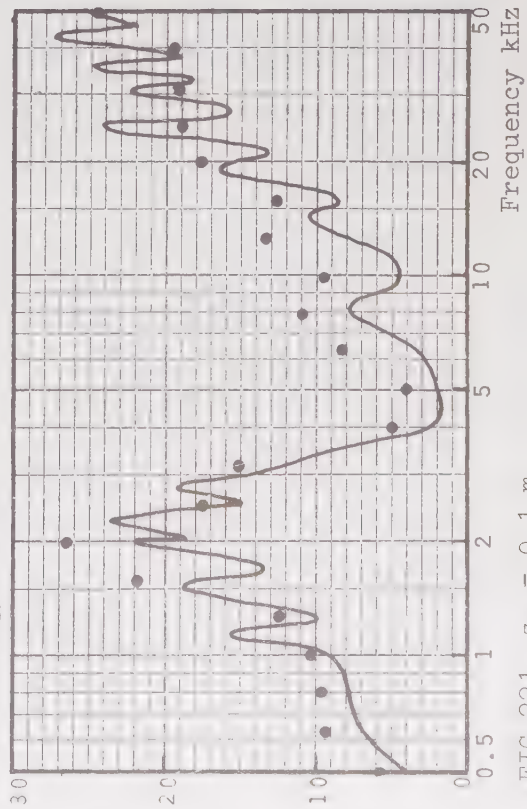


FIG. 221. $z_R = 0.1 \text{ m}$

FIG. 218. $z_R = 0.6 \text{ m}$

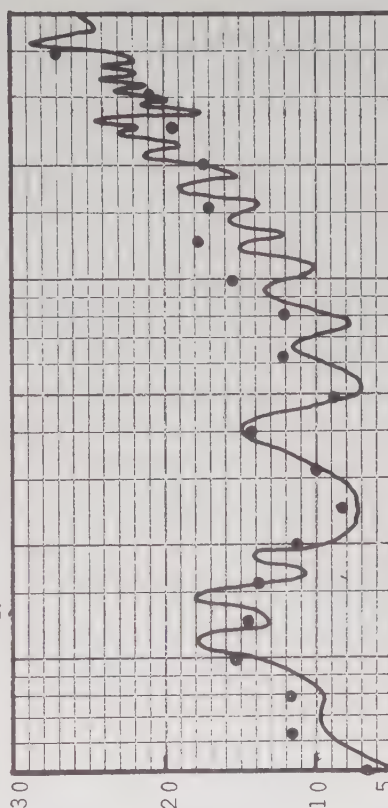


FIG. 220. $z_R = 0.2 \text{ m}$

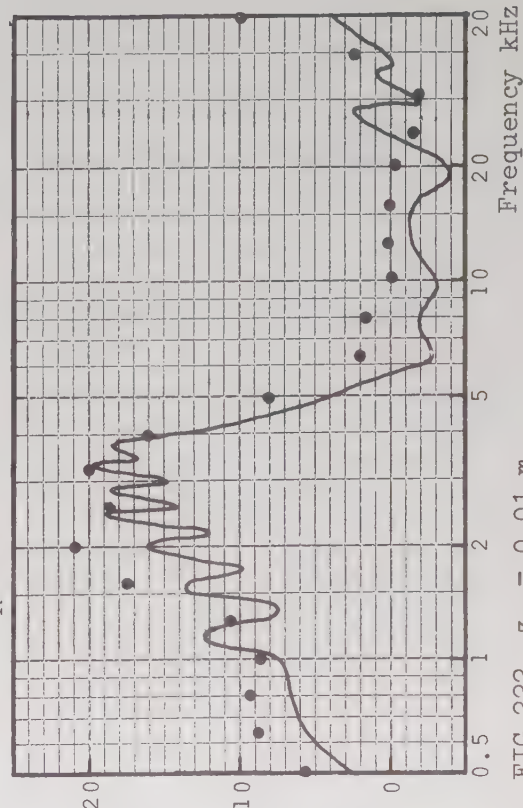


FIG. 222. $z_R = 0.01 \text{ m}$

Frequency kHz

S E C T I O N 3

OUTDOOR EXPERIMENTS

The following figures, 301-379, have been obtained from the outdoor experiments.

Ground impedance: Type, see head of each page, cf. Eq. (5). The following types have been examined:

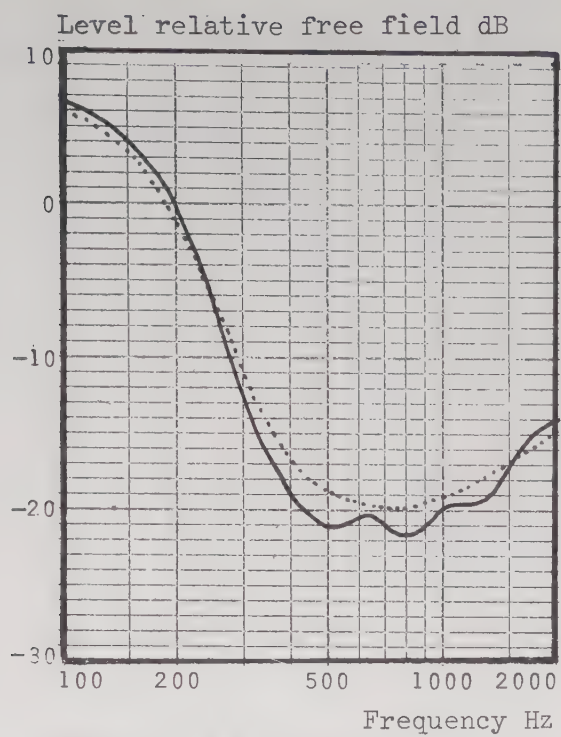
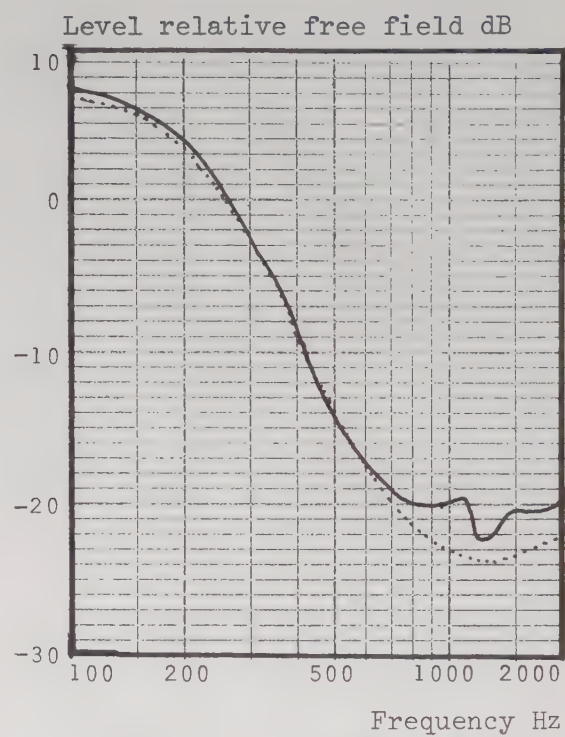
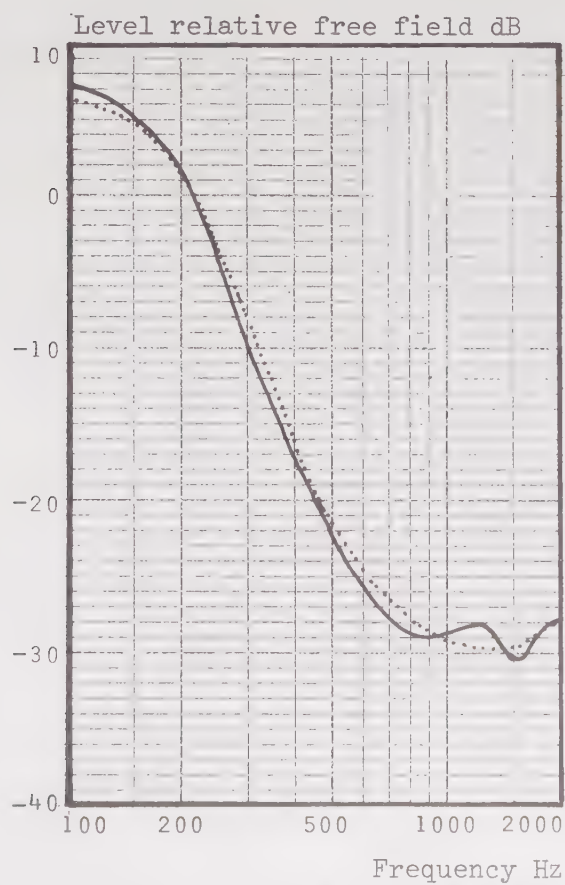
Stubble-field: $a = 0.6$, $b = 10^0$, $c = 12000$ Hz, $d = 500$ Hz;
 New-sown: $a = 0.6$, $b = 10^0$, $c = 30000$ Hz, $d = 1000$ Hz;
 Grassland: $a = 0.25$, $b = 10^0$, $c = 7500$ Hz, $d = 425$ Hz.

Geometry: See head of each page and compare with Fig. 7. The vertical distance source-receiver is $r = [(x_S - x_R)^2 + (y_S - y_R)^2]^{1/2}$.

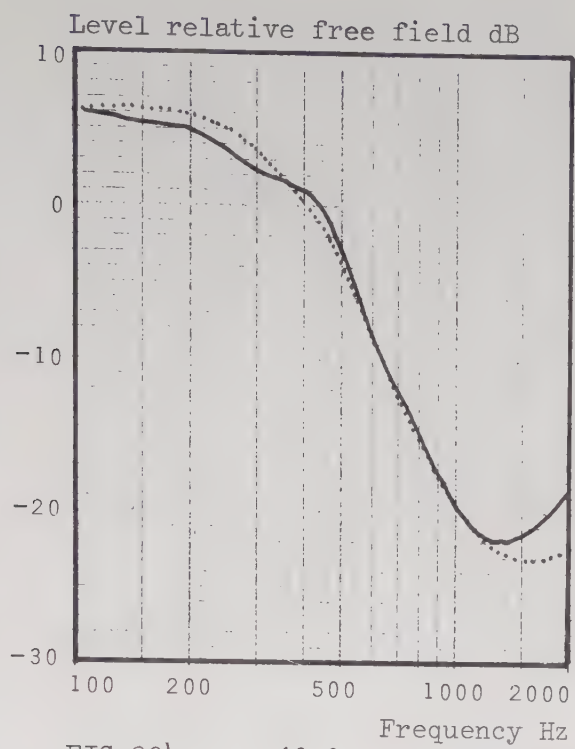
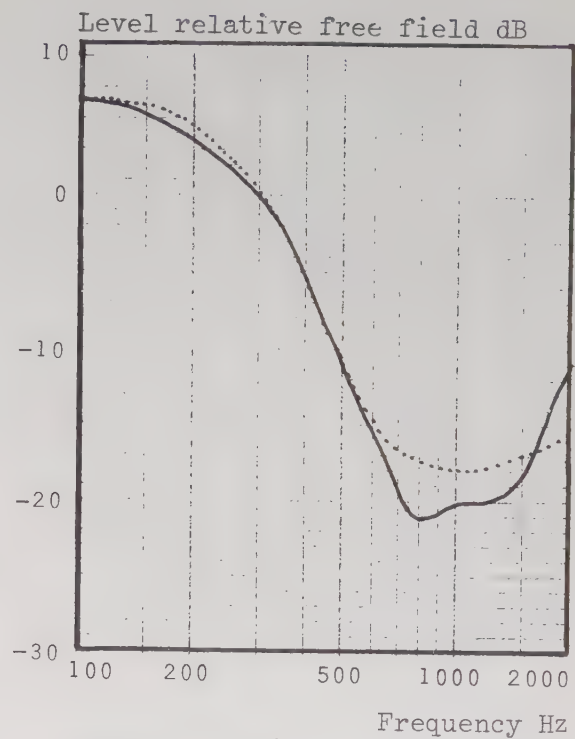
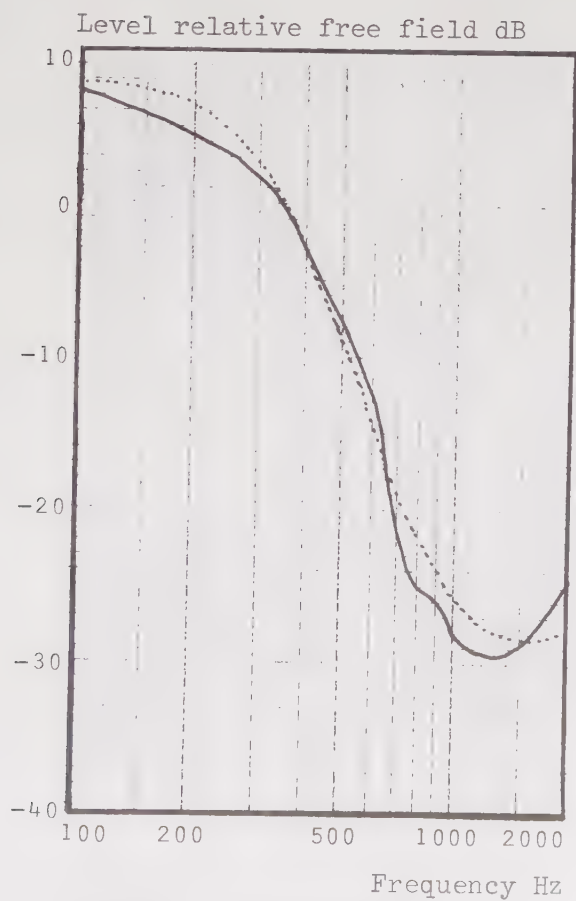
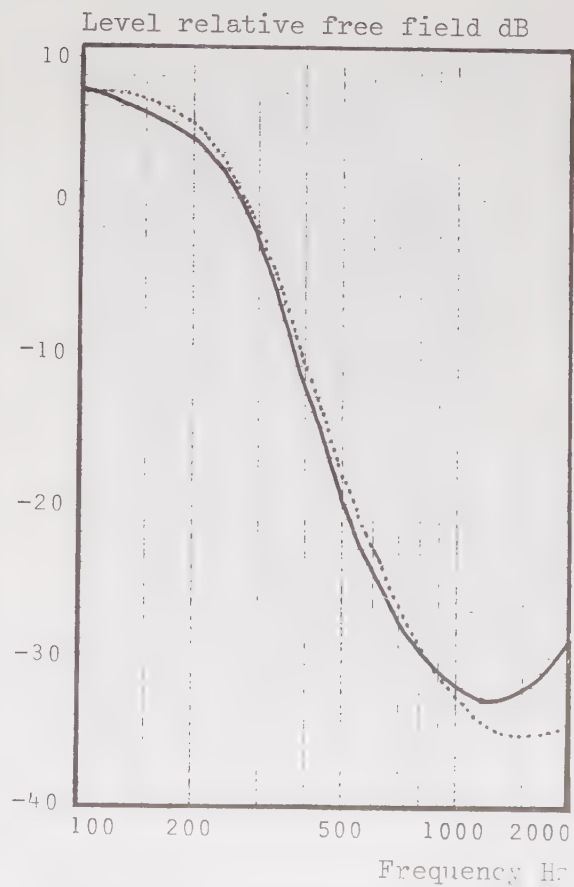
Calculations: Ground without screen, Figs. 301-335. Calculated with Eqs. (5) and (8)-(15) via subroutine FIELD (Part II).
 Ground with screen, Figs. 336-379. Calculated with Eqs. (5), (8)-(15), (25) and (33) via subroutine INTEG (Part II).

Notation: ——— measured,
 ----- calculated.

$$z_S = 0.1 \text{ m}$$

FIG.301. $r = 20.0 \text{ m}$, $z_R = 0.5 \text{ m}$ FIG.302. $r = 10.0 \text{ m}$, $z_R = 0.1 \text{ m}$ FIG.303. $r = 20.0 \text{ m}$, $z_R = 0.1 \text{ m}$

$$z_S = 0.1 \text{ m}$$

FIG.304. $r = 10.0 \text{ m}$, $z_R = 0.1 \text{ m}$ FIG.305. $r = 40.0 \text{ m}$, $z_R = 0.5 \text{ m}$ FIG.306. $r = 20.0 \text{ m}$, $z_R = 0.1 \text{ m}$ FIG.307. $r = 40.0 \text{ m}$, $z_R = 0.1 \text{ m}$

$$r = 20.0 \text{ m} \quad z_S = 0.1 \text{ m} \quad z_R = 0.1 \text{ m}$$

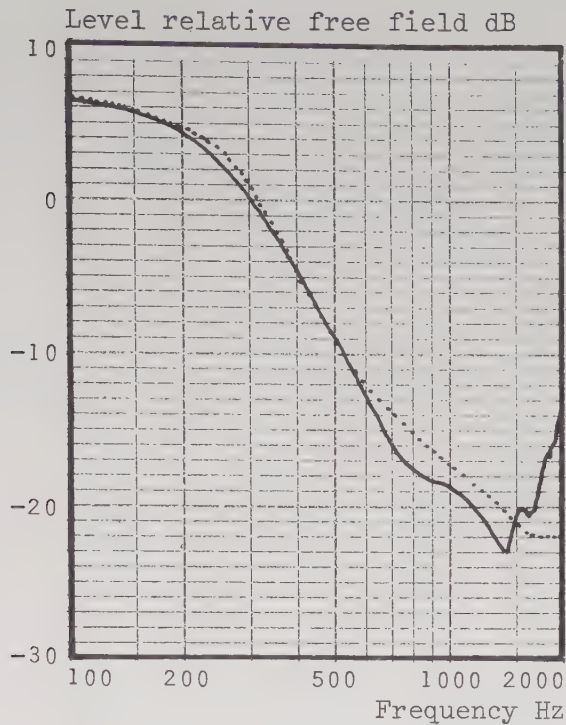


FIG.308. From the series in Figs.312-335.

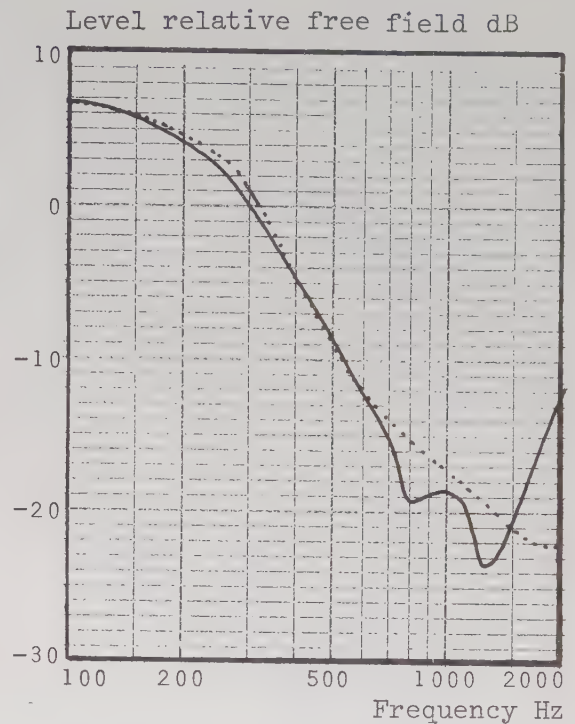


FIG.309. Measured three hours later. Same position as in Fig. 308.

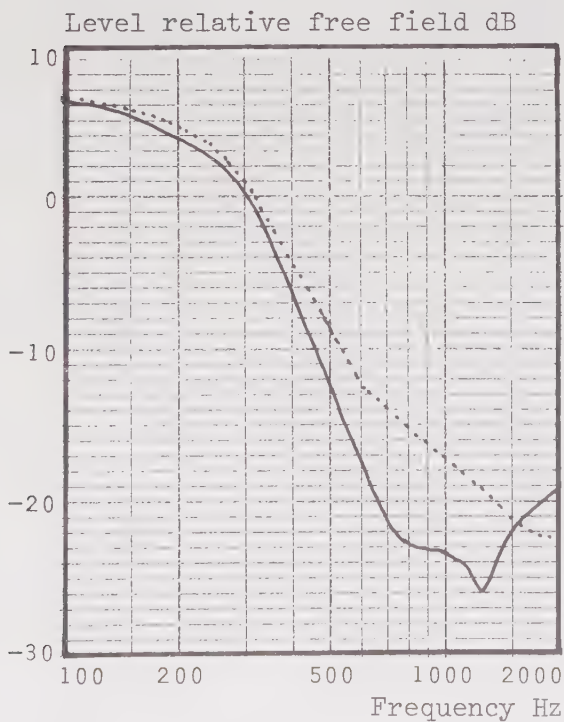


FIG.310. Measured three hours later and in a direction with a right angle to the one in Fig.308.

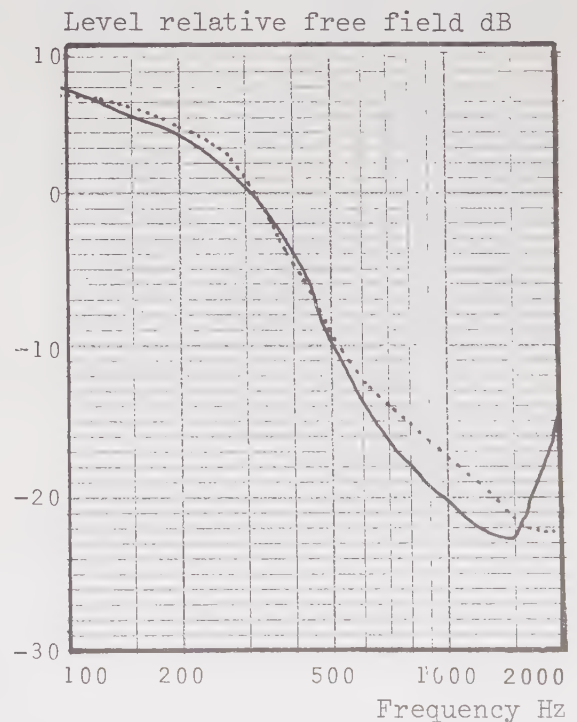
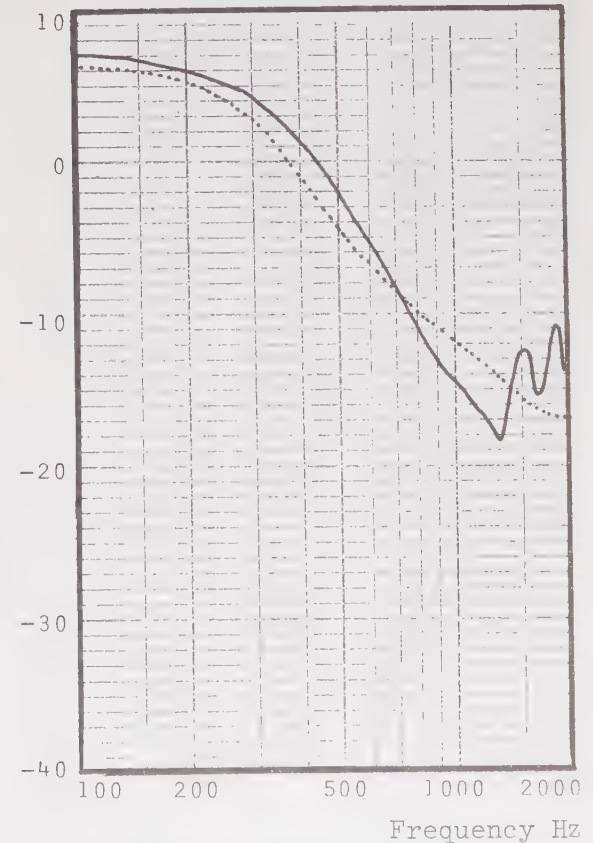
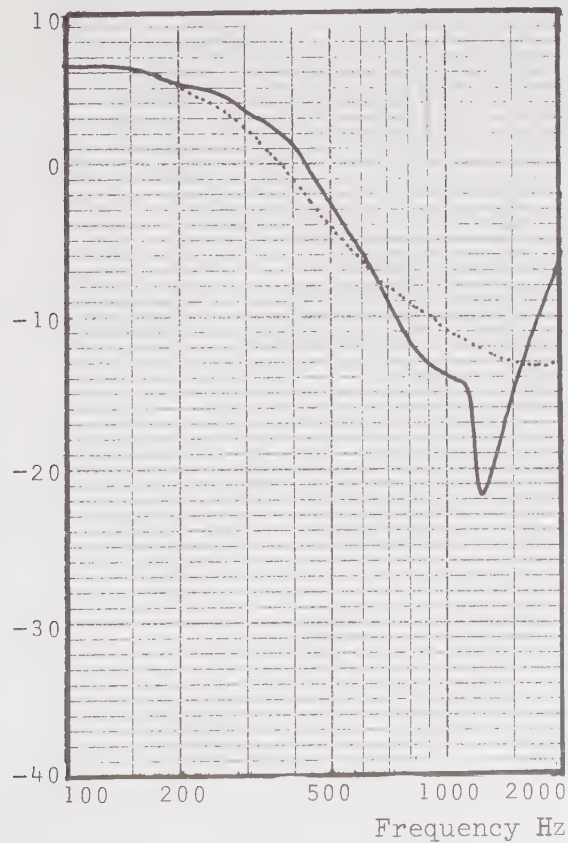
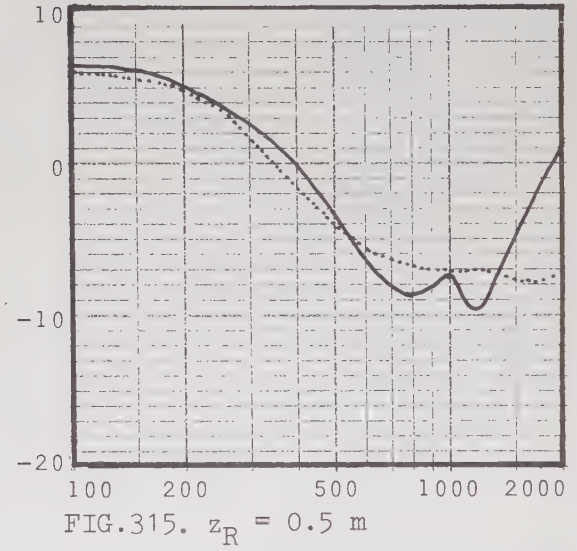
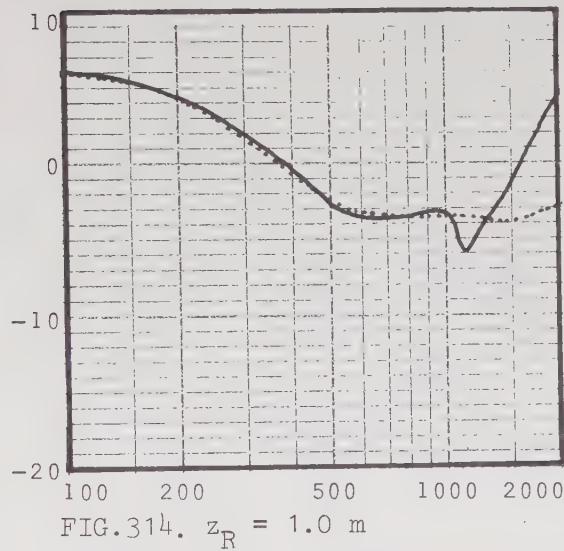
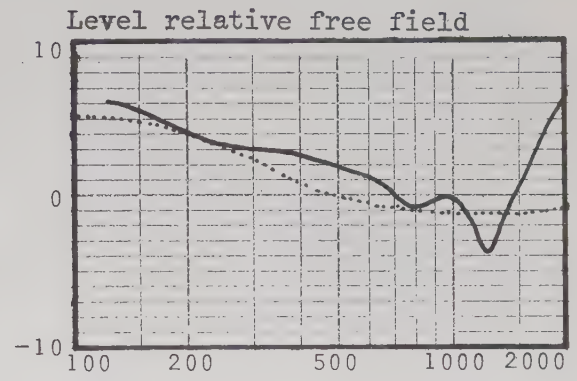
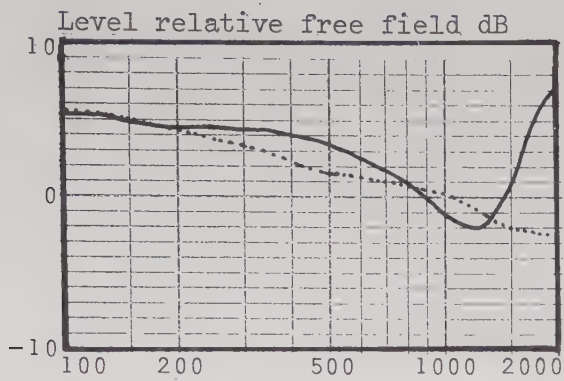
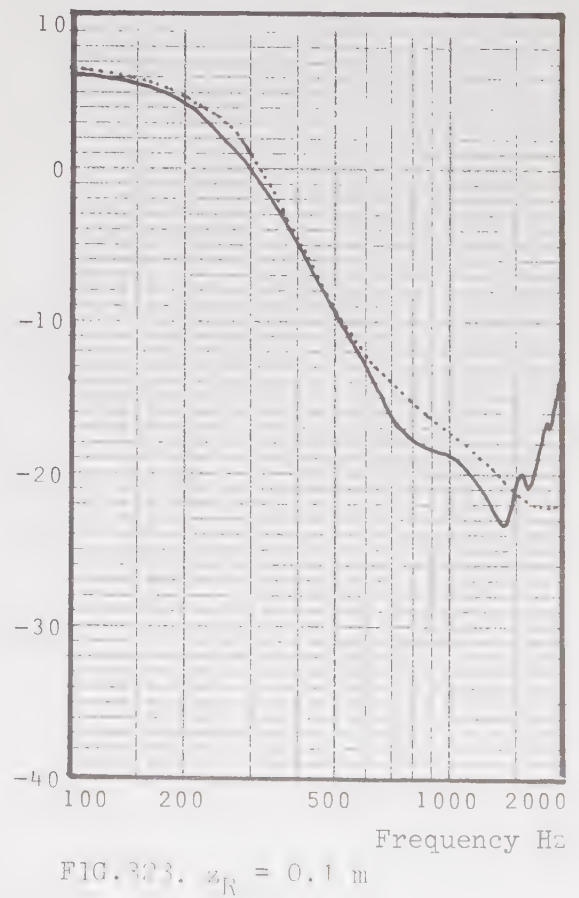
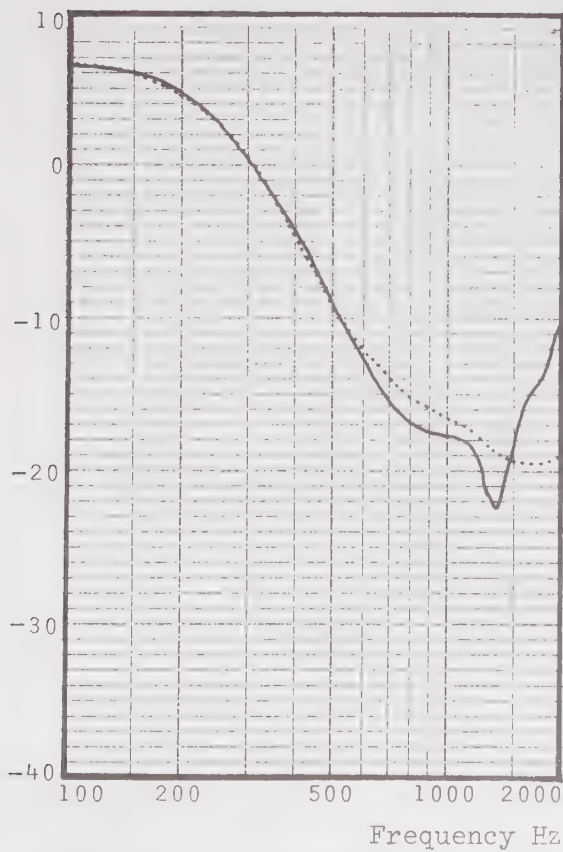
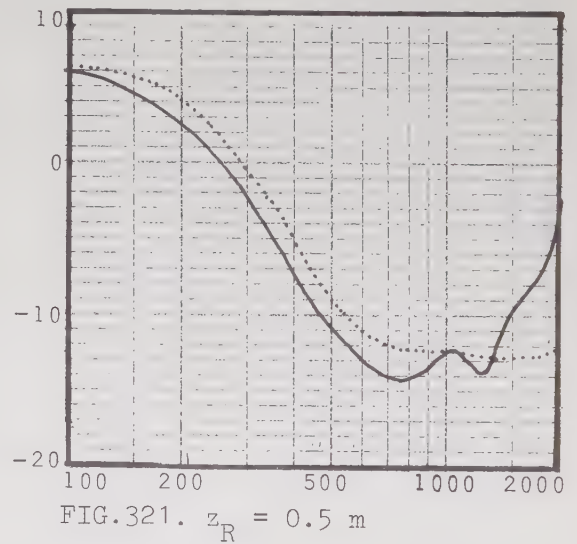
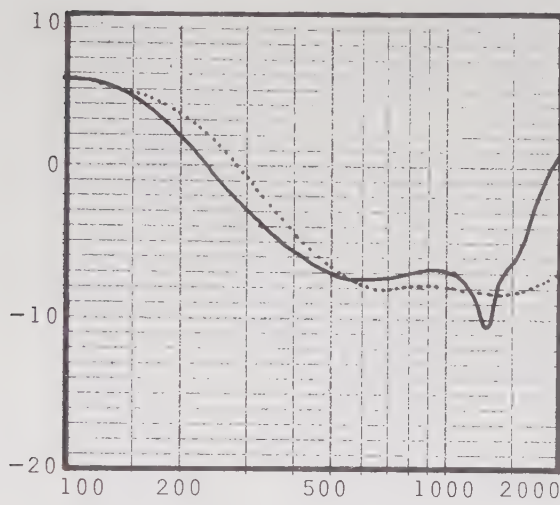
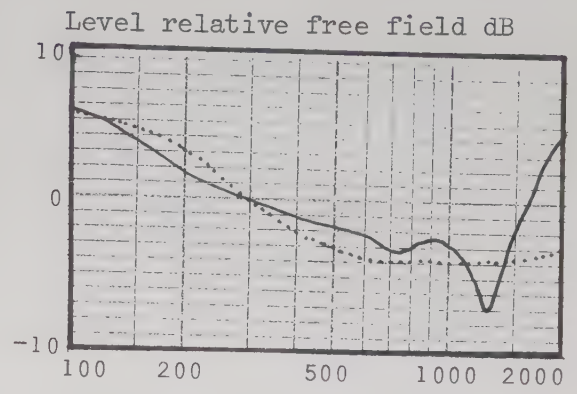
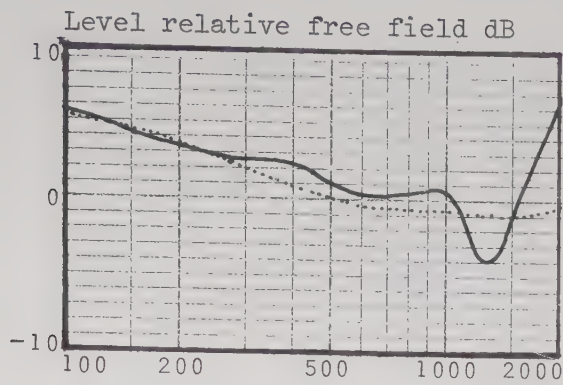


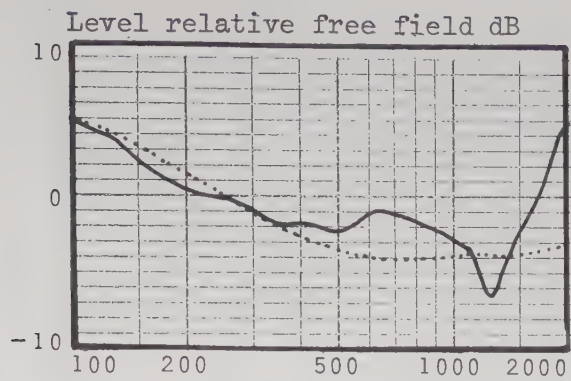
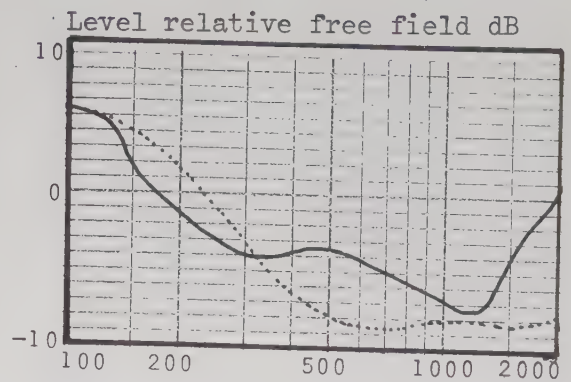
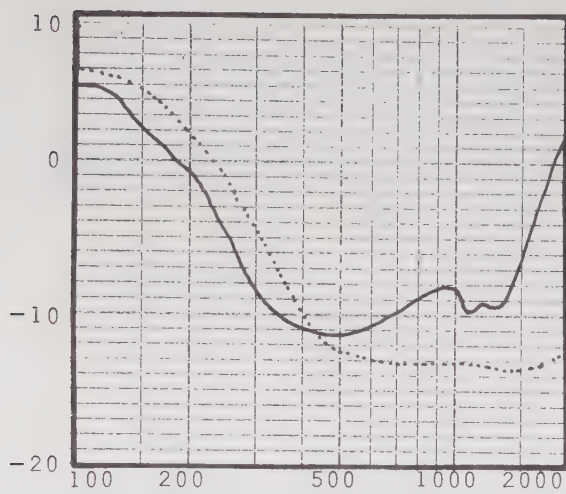
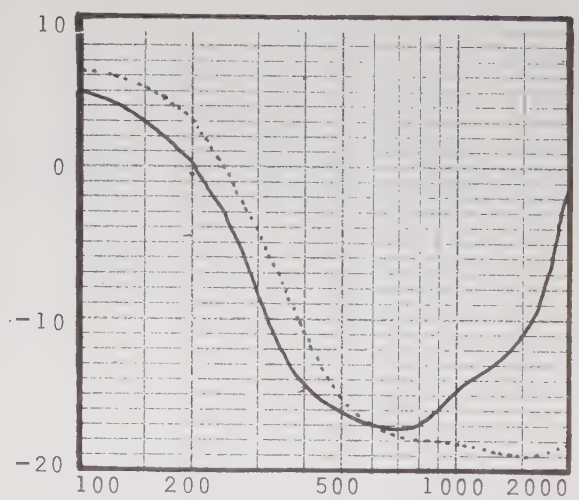
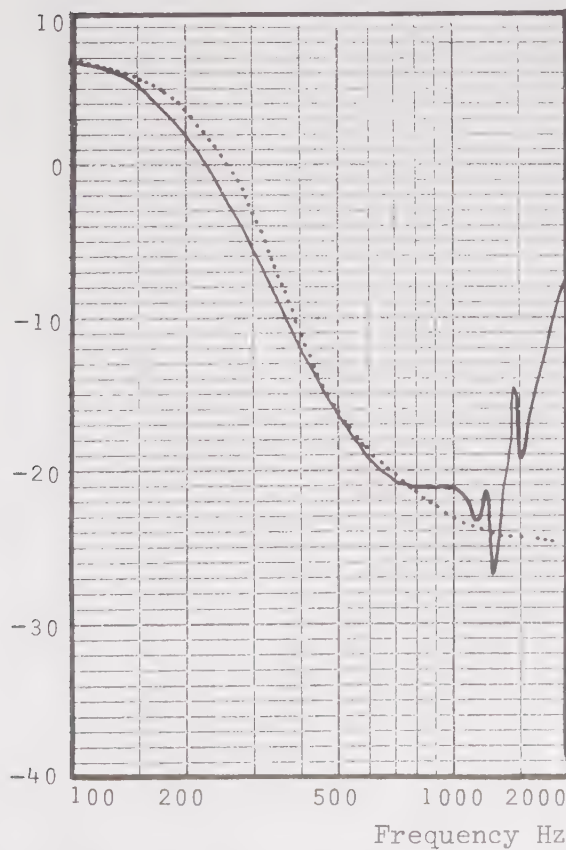
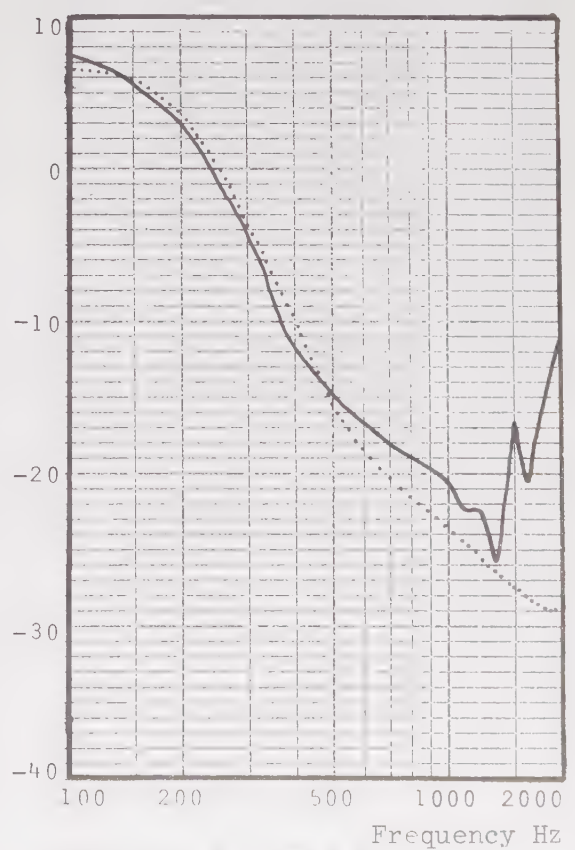
FIG.311. Measured three days later. New position relative Fig.308.

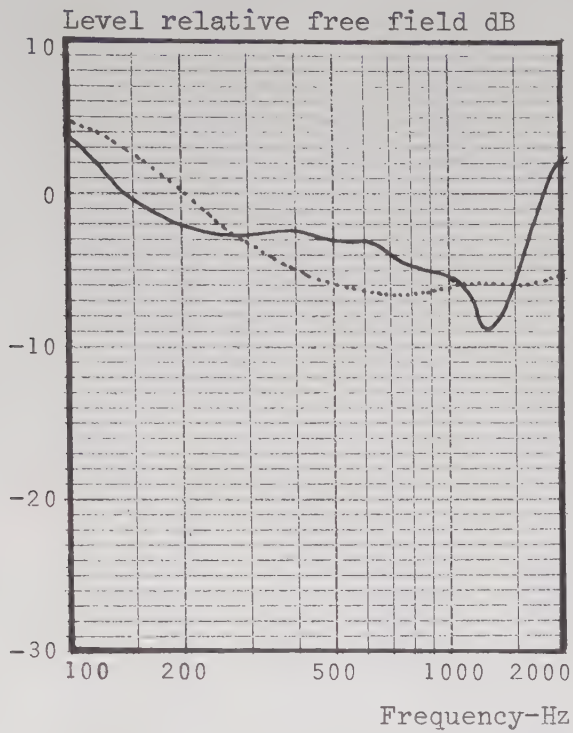
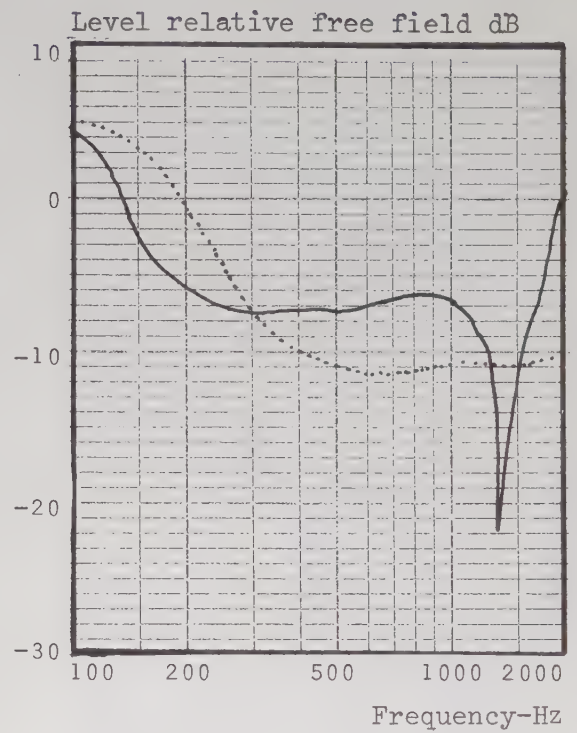
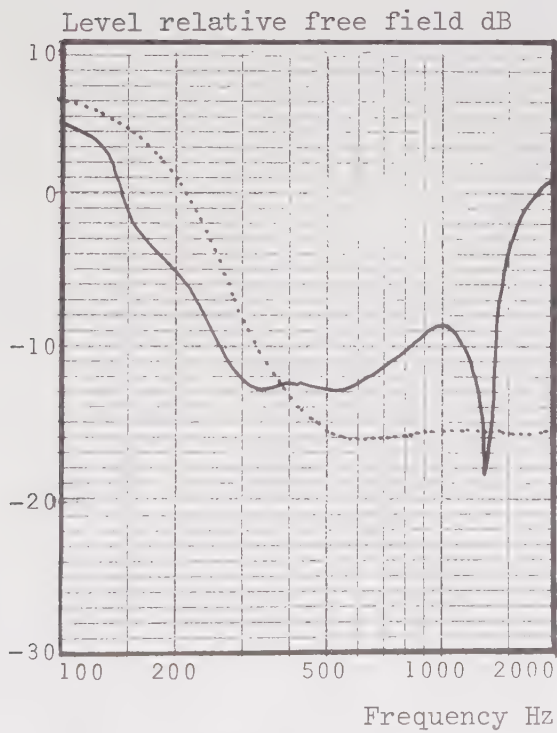
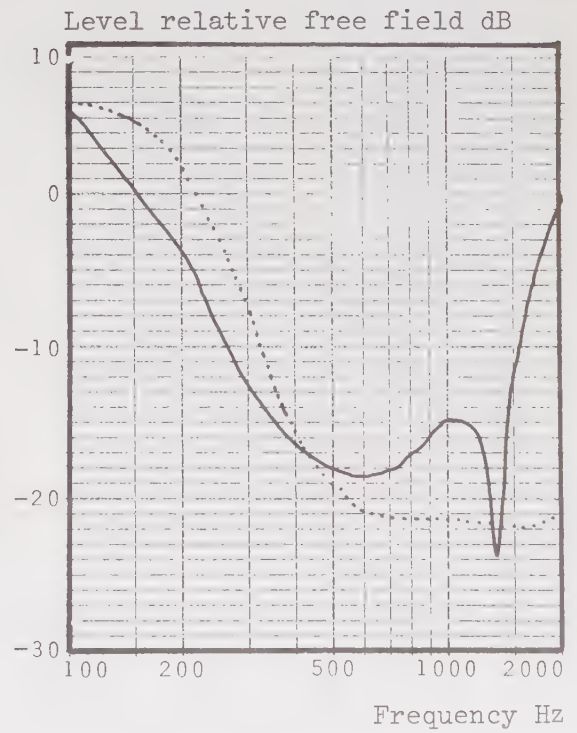
$r = 10.0 \text{ m}$ $z_S = 0.1 \text{ m}$ 

$r = 20.0 \text{ m}$ $z_S = 0.1 \text{ m}$ 

$$r = 40.0 \text{ m}$$

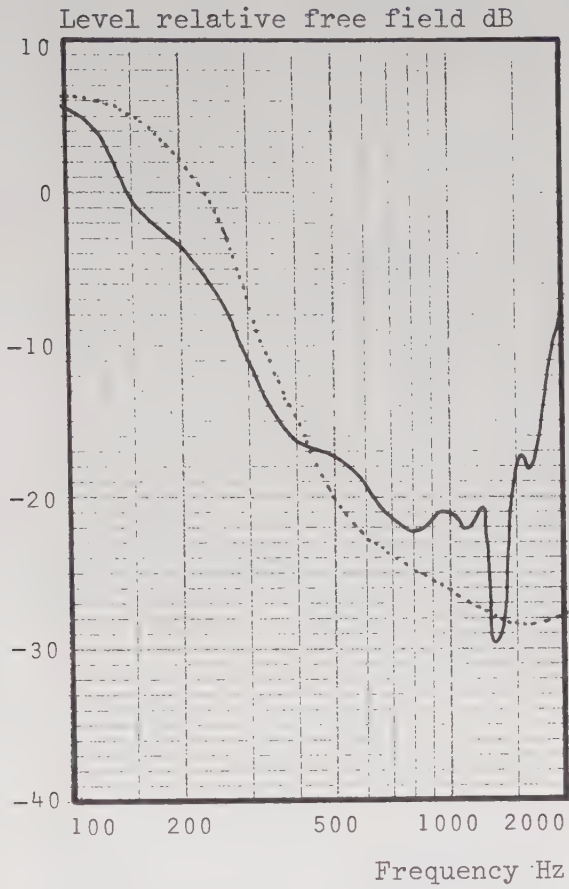
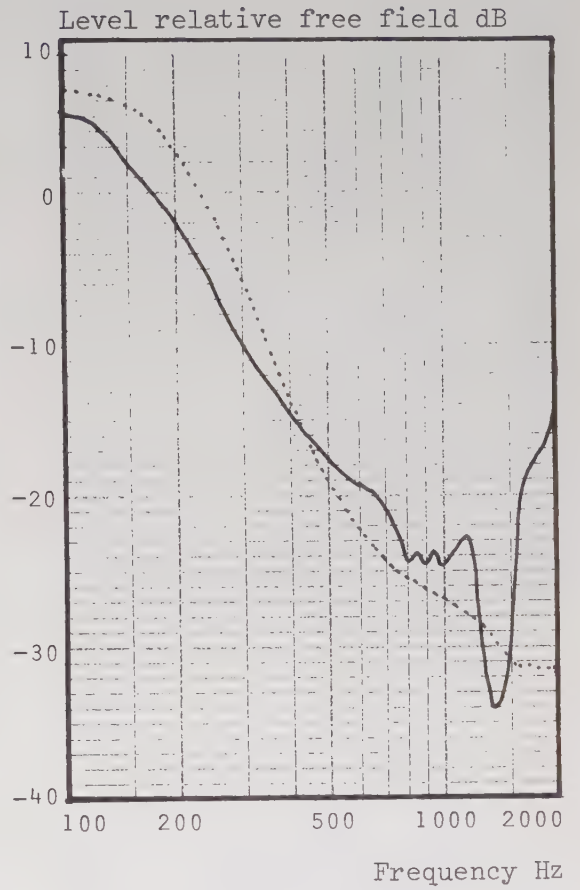
$$z_S = 0.1 \text{ m}$$

FIG.324. $z_R = 3.7 \text{ m}$ FIG.325. $z_R = 2.0 \text{ m}$ FIG.326. $z_R = 1.0 \text{ m}$ FIG.327. $z_R = 0.5 \text{ m}$ FIG.328. $z_R = 0.2 \text{ m}$ FIG.329. $z_R = 0.1 \text{ m}$

$r = 60.0 \text{ m}$ $z_S = 0.1 \text{ m}$ FIG.330. $z_R = 3.7 \text{ m}$ FIG.331. $z_R = 2.0 \text{ m}$ FIG.332. $z_R = 1.0 \text{ m}$ FIG.333. $z_R = 0.5 \text{ m}$

$$r = 60.0 \text{ m}$$

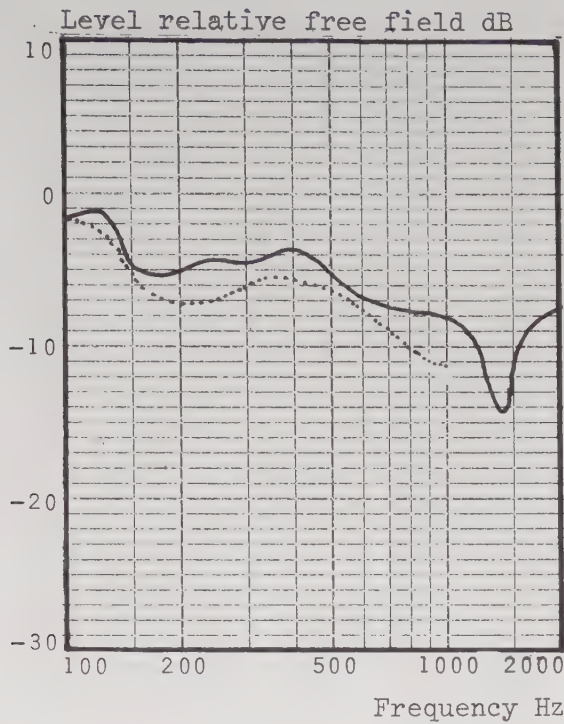
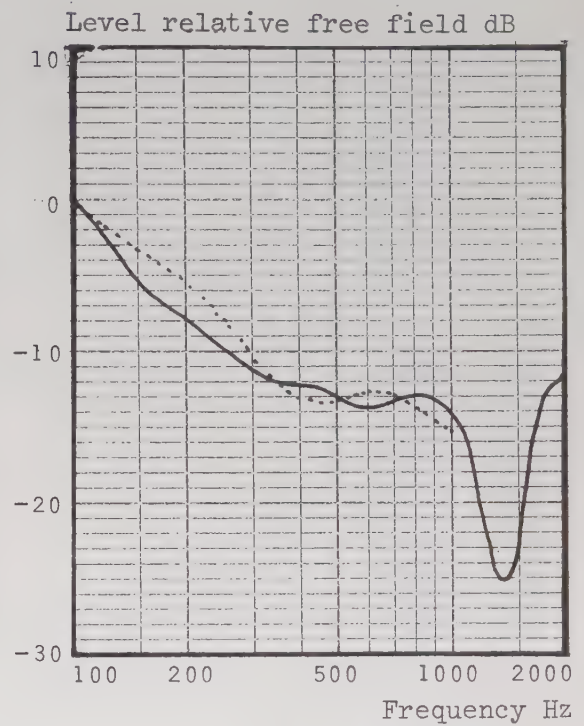
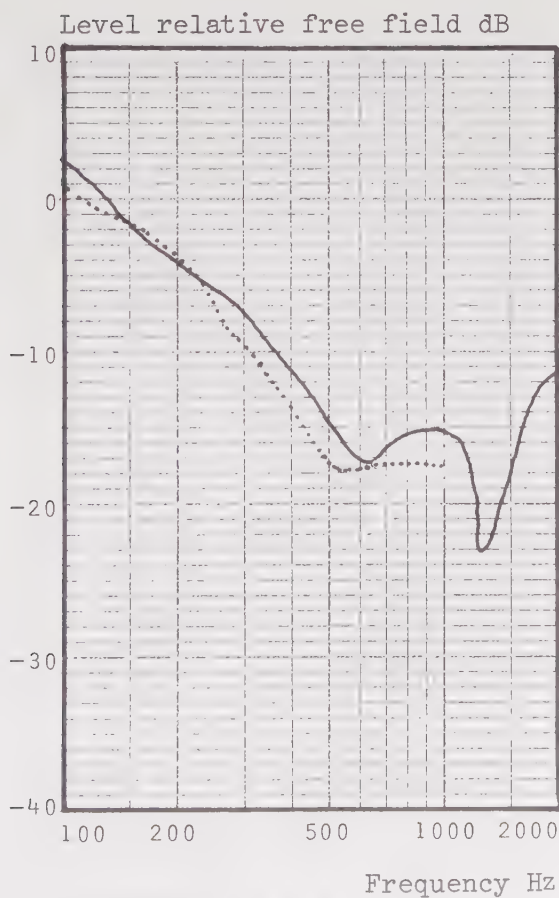
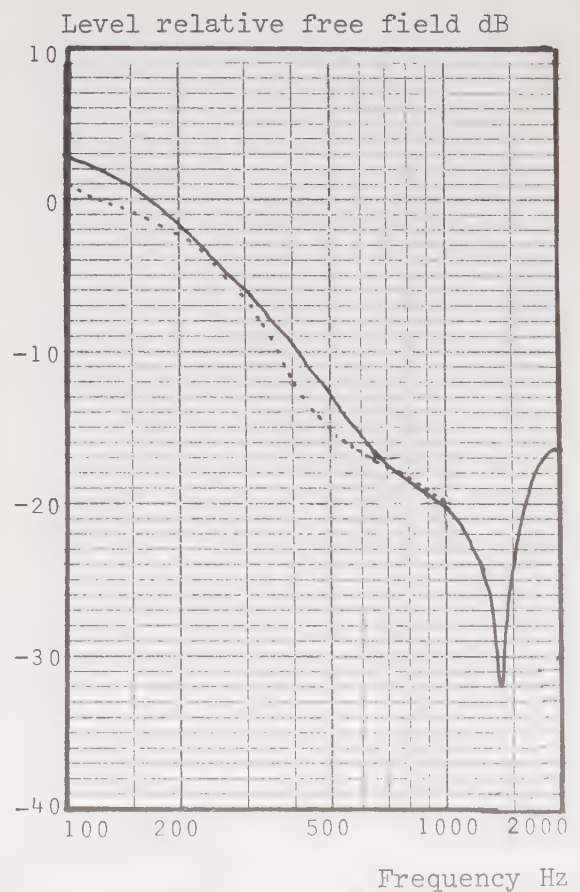
$$z_G = 0.1 \text{ m}$$

FIG.334. $z_R = 0.2 \text{ m}$ FIG.335. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

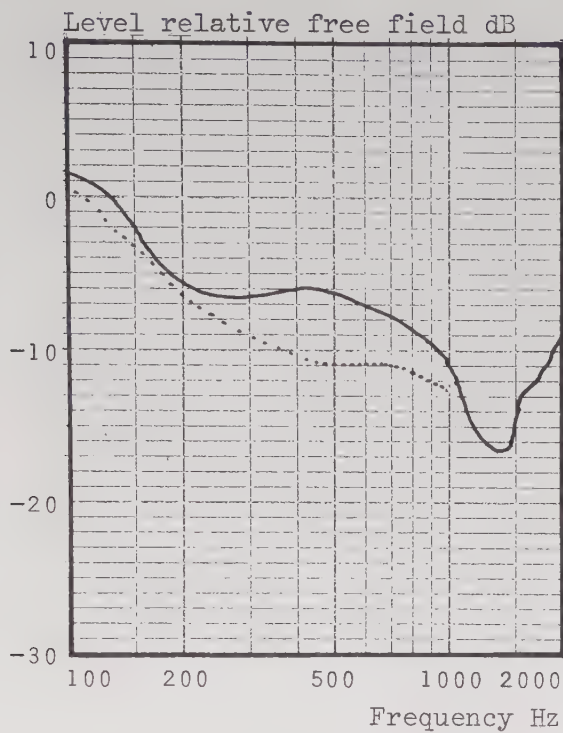
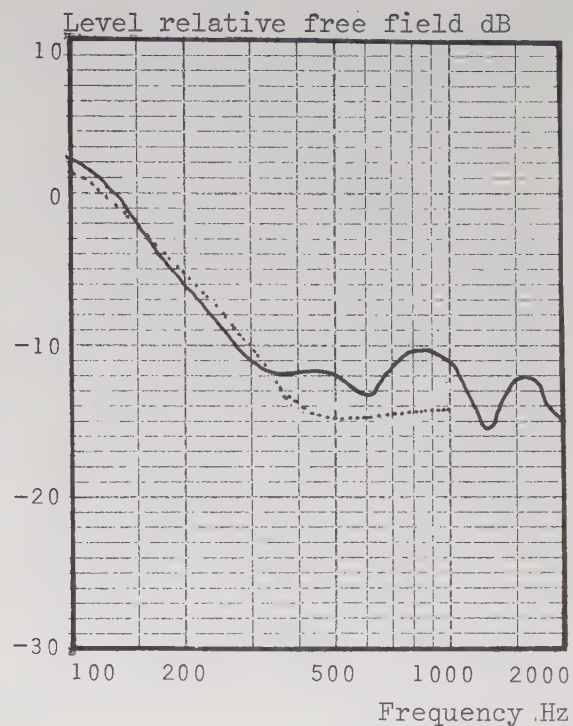
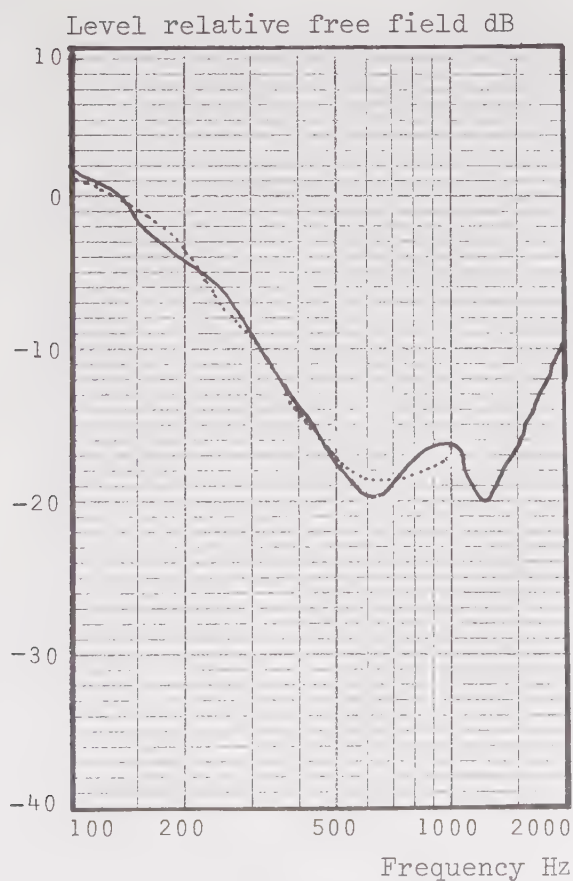
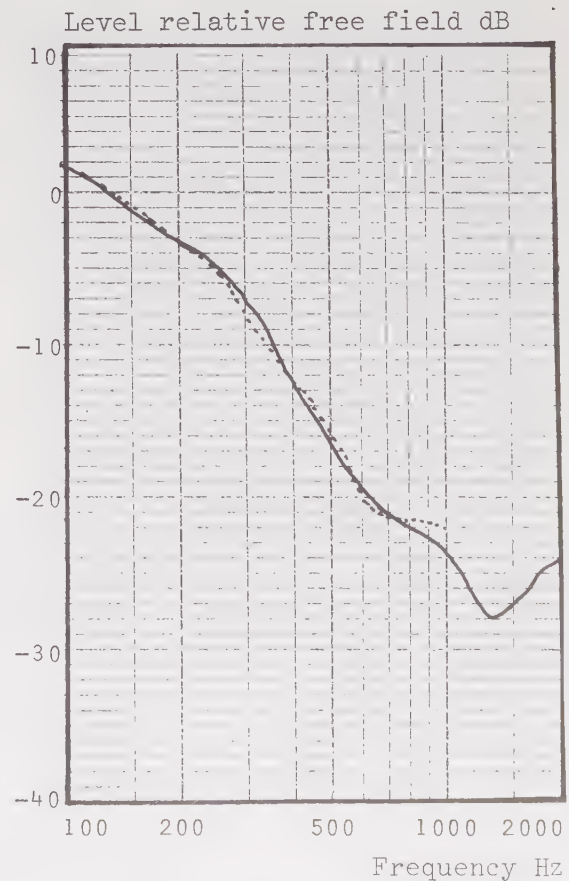
$$(x_S, y_S, z_S) = (10, 0, 0.1) \text{ m}$$

$$(x_R, y_R, z_R) = (-10, 0, z_R) \text{ m}$$

FIG.336. $z_R = 2.0 \text{ m}$ FIG.337. $z_R = 1.0 \text{ m}$ FIG.338. $z_R = 0.5 \text{ m}$ FIG.339. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

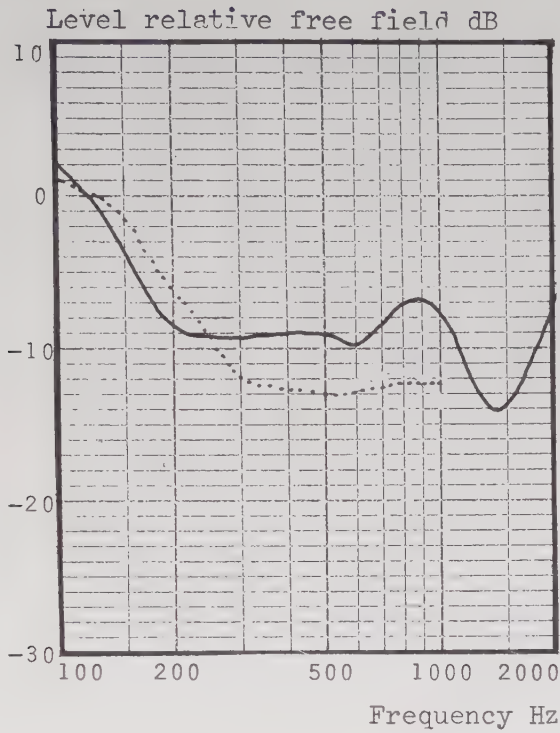
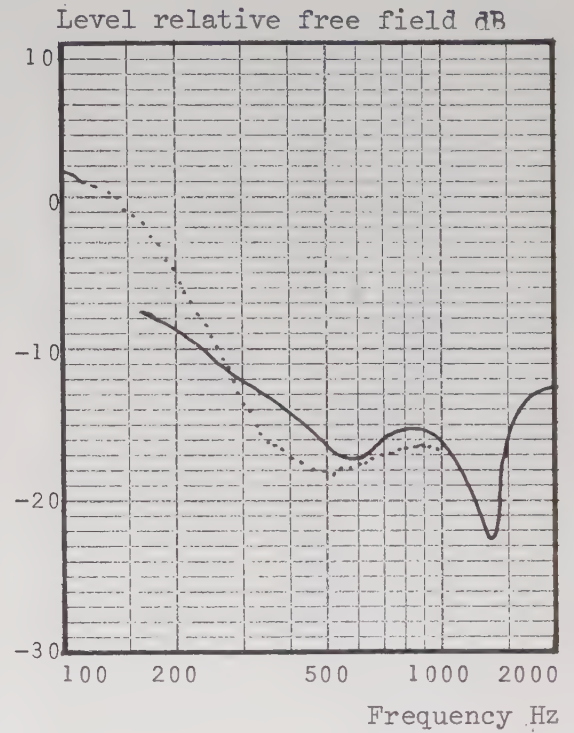
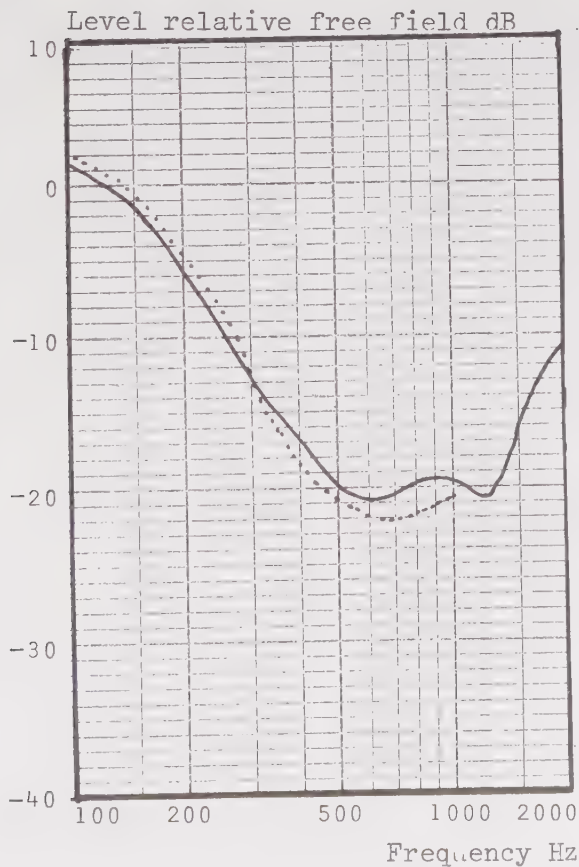
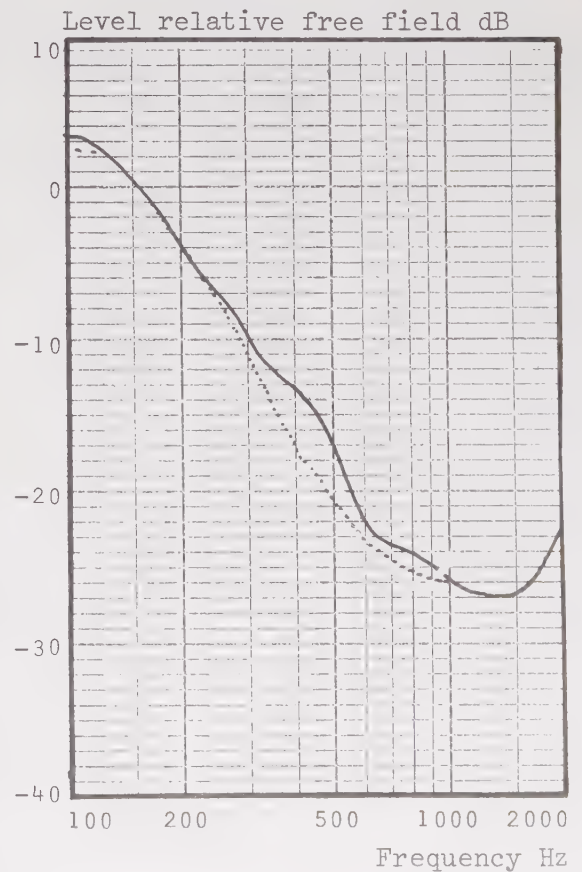
$$(x_S, y_S, z_S) = (10, 0, 0.1) \text{ m} \quad (x_R, y_R, z_R) = (-20, 0, z_R) \text{ m}$$

FIG.340. $z_R = 2.0 \text{ m}$ FIG.341. $z_R = 1.0 \text{ m}$ FIG.342. $z_R = 0.5 \text{ m}$ FIG.343. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (10, 0, 0.1) \text{ m}$$

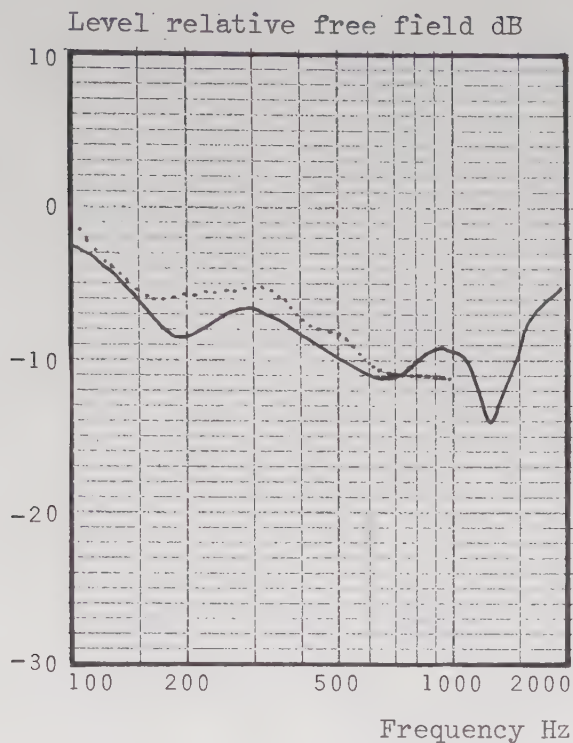
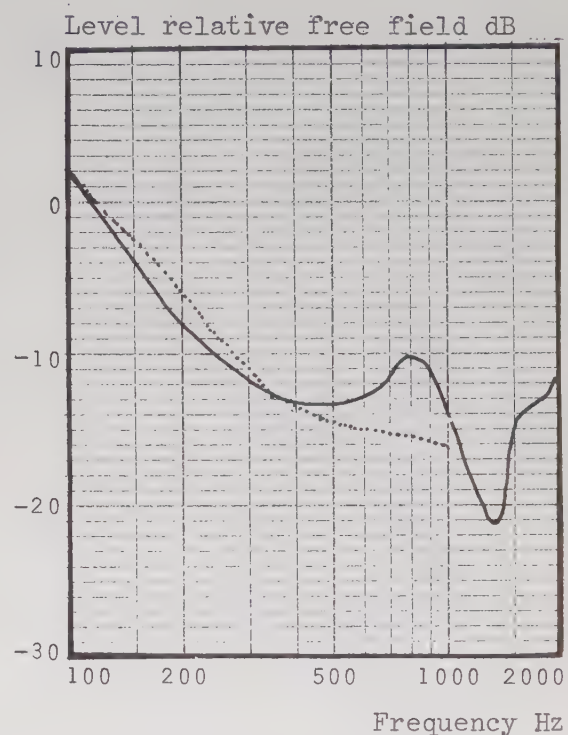
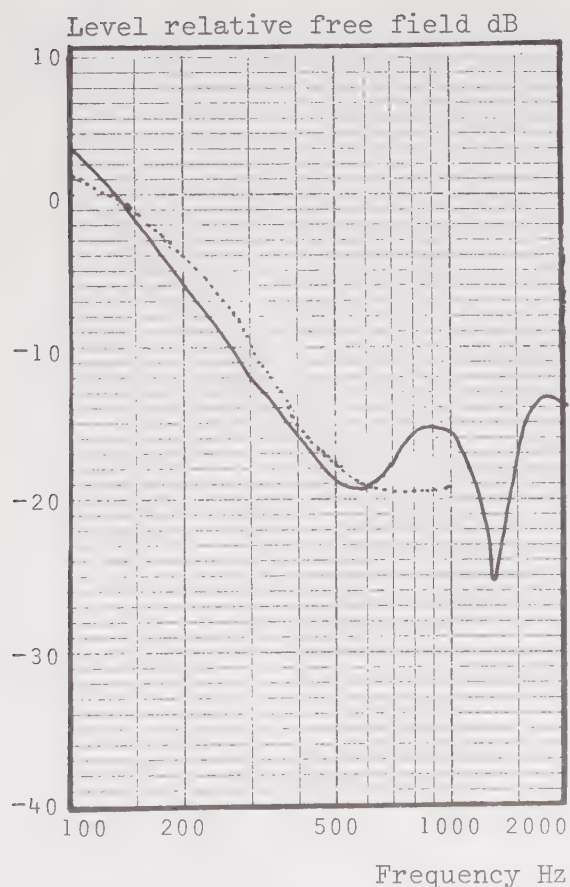
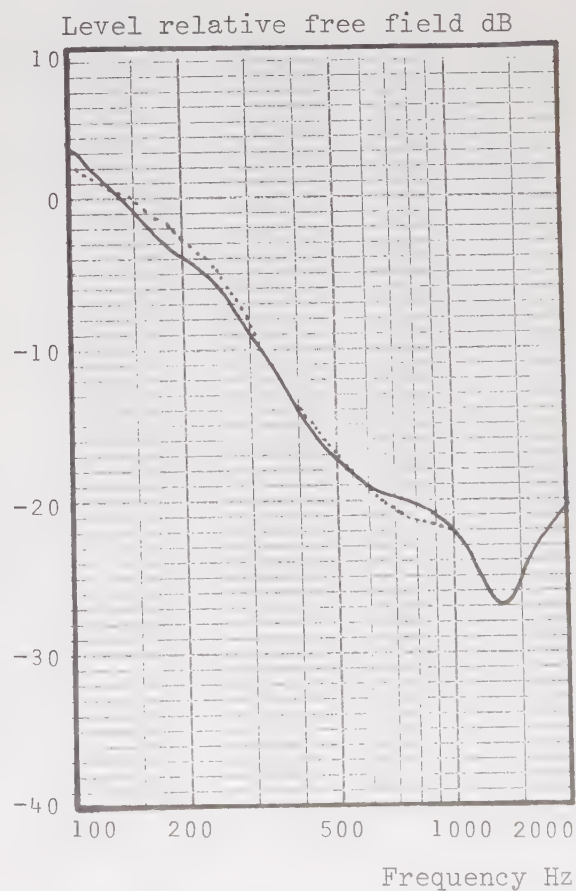
$$(x_R, y_R, z_R) = (-40, 0, z_R) \text{ m}$$

FIG.344. $z_R = 2.0 \text{ m}$ FIG.345. $z_R = 1.0 \text{ m}$ FIG.346. $z_R = 0.5 \text{ m}$ FIG.347. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (20, 0, 0.1) \text{ m}$$

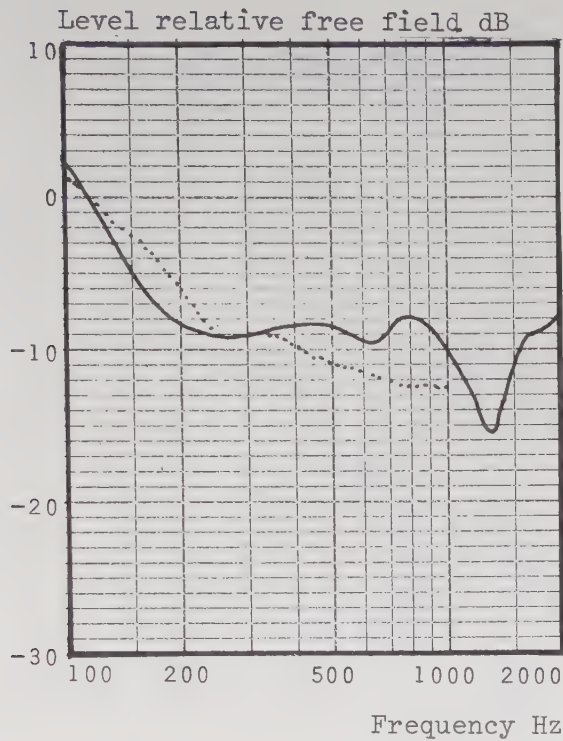
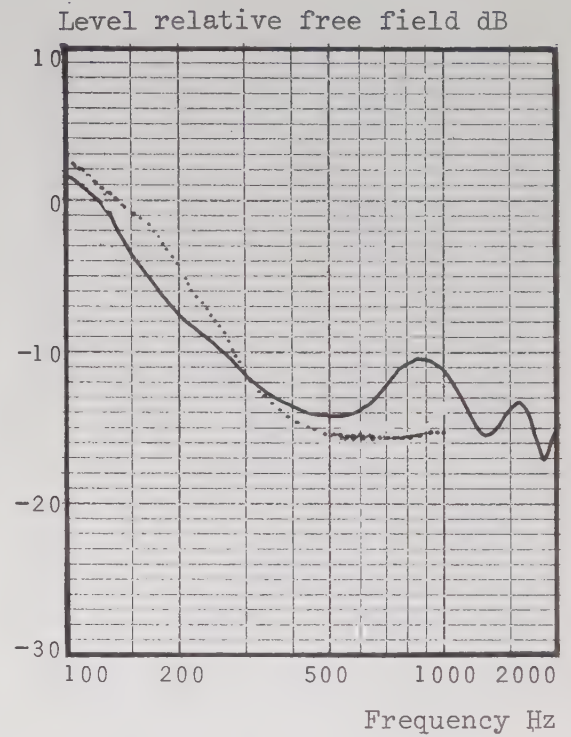
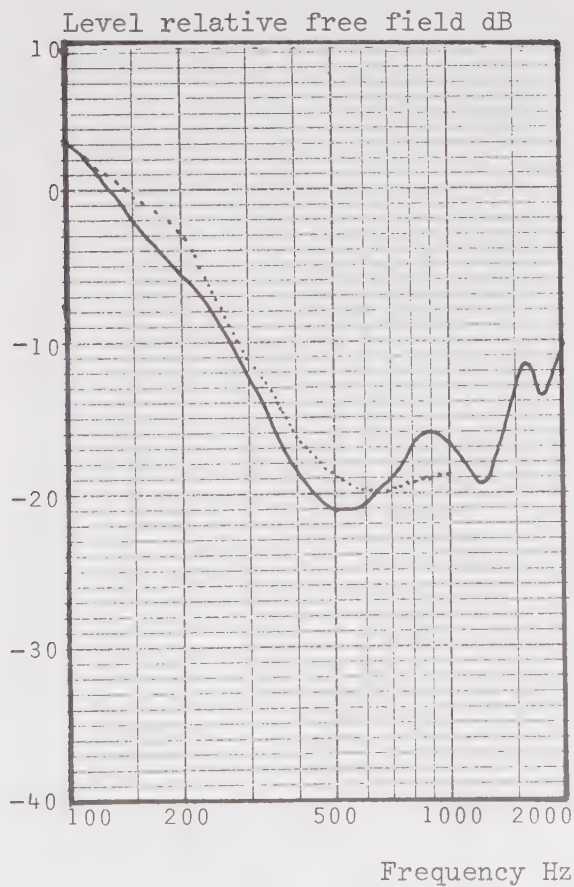
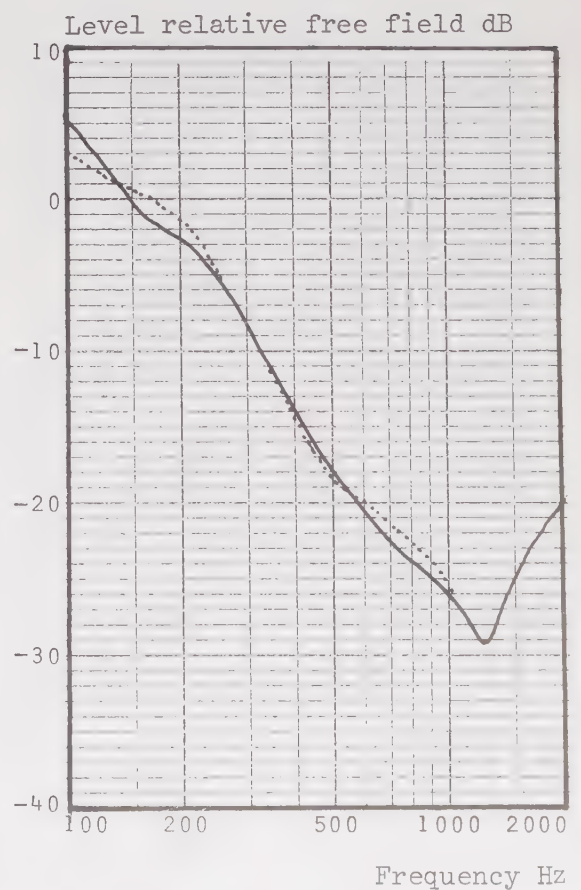
$$(x_R, y_R, z_R) = (-10, 0, z_R) \text{ m}$$

FIG.348. $z_R = 2.0 \text{ m}$ FIG.349. $z_R = 1.0 \text{ m}$ FIG.350. $z_R = 0.5 \text{ m}$ FIG.351. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (20, 0, 0.1) \text{ m}$$

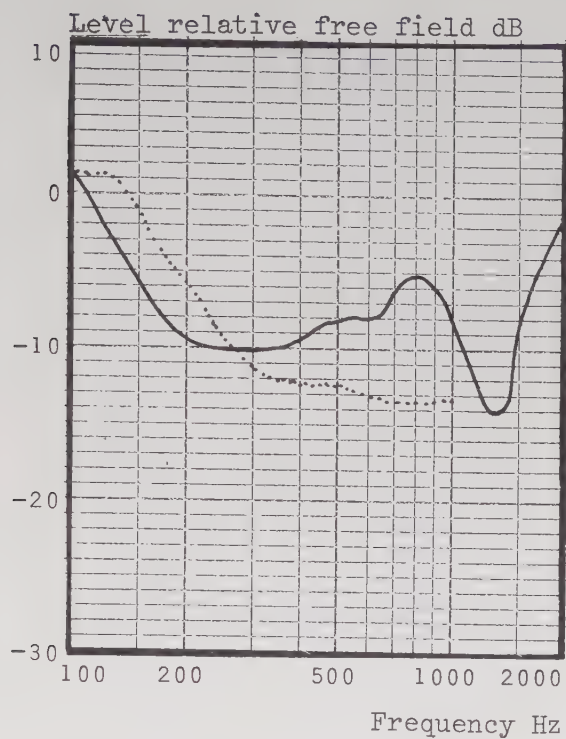
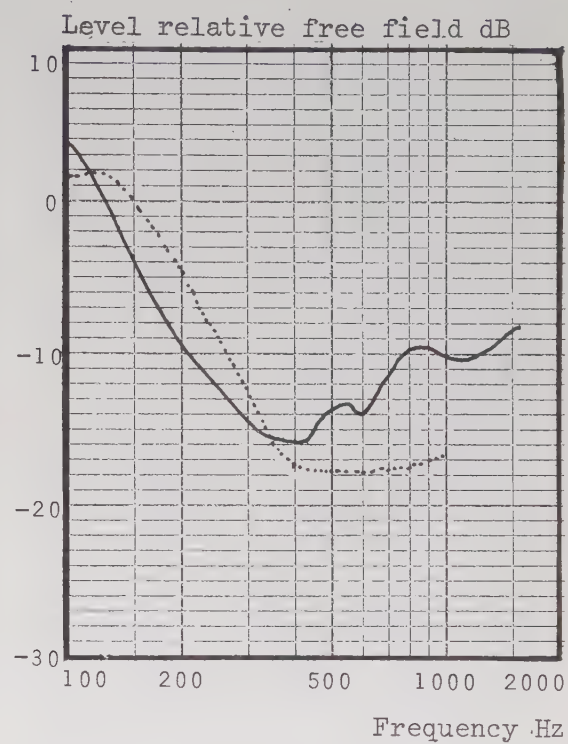
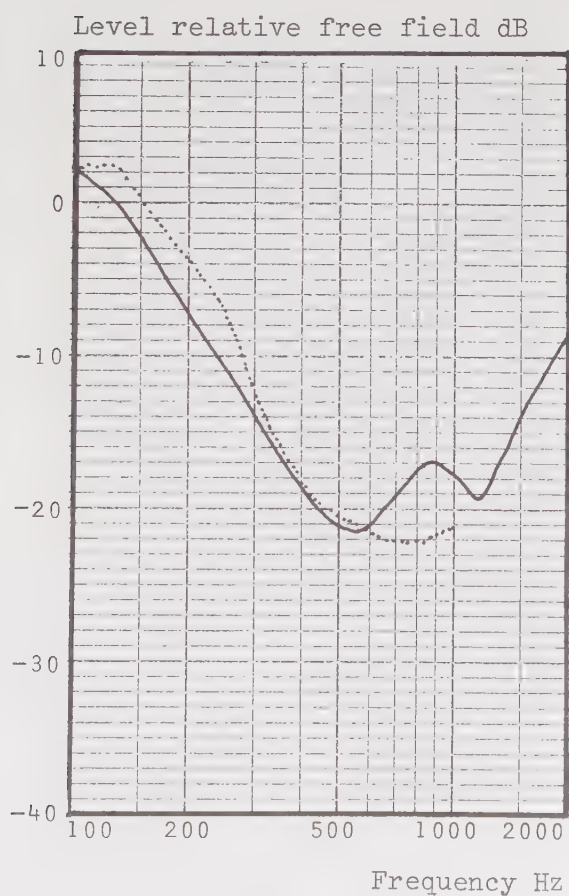
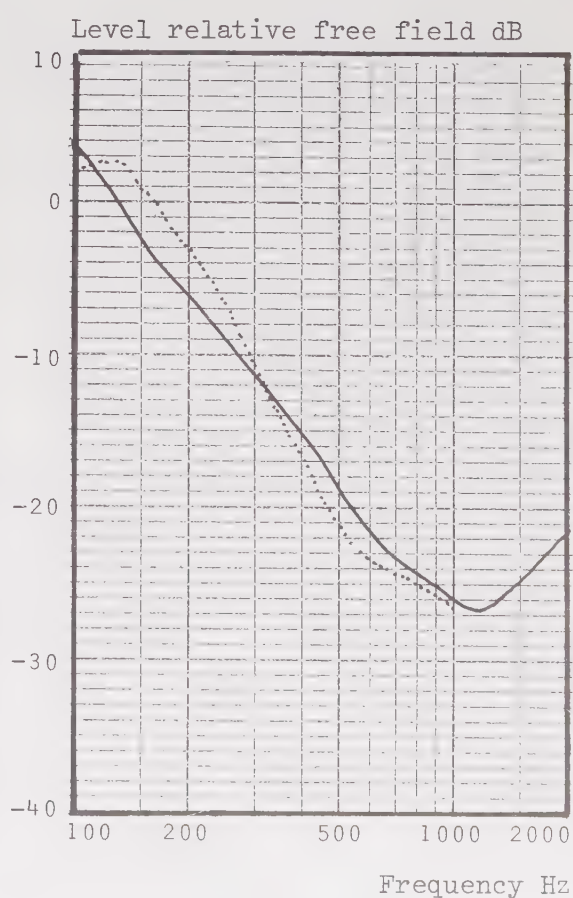
$$(x_R, y_R, z_R) = (-20, 0, z_R) \text{ m}$$

FIG.352. $z_R = 2.0 \text{ m}$ FIG.353. $z_R = 1.0 \text{ m}$ FIG.354. $z_R = 0.5 \text{ m}$ FIG.355. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (20, 0, 0.1) \text{ m}$$

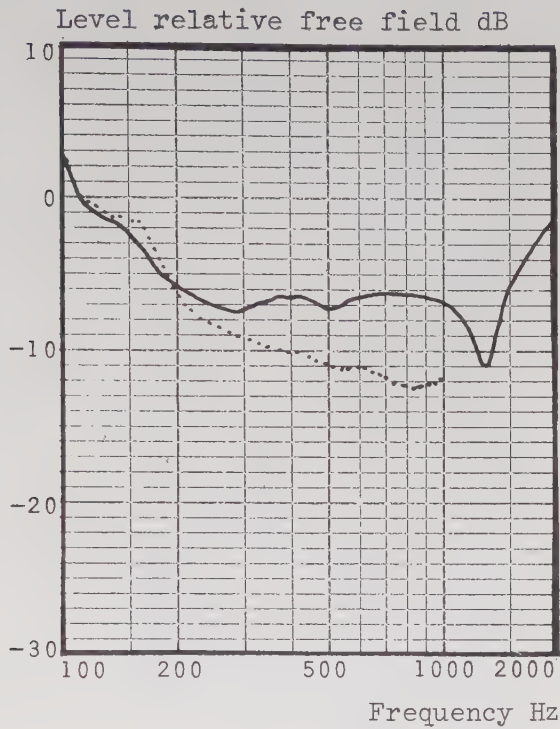
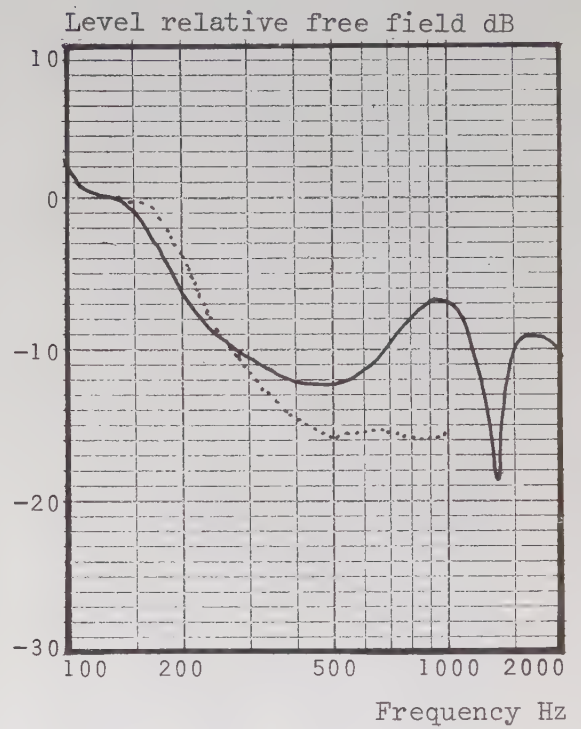
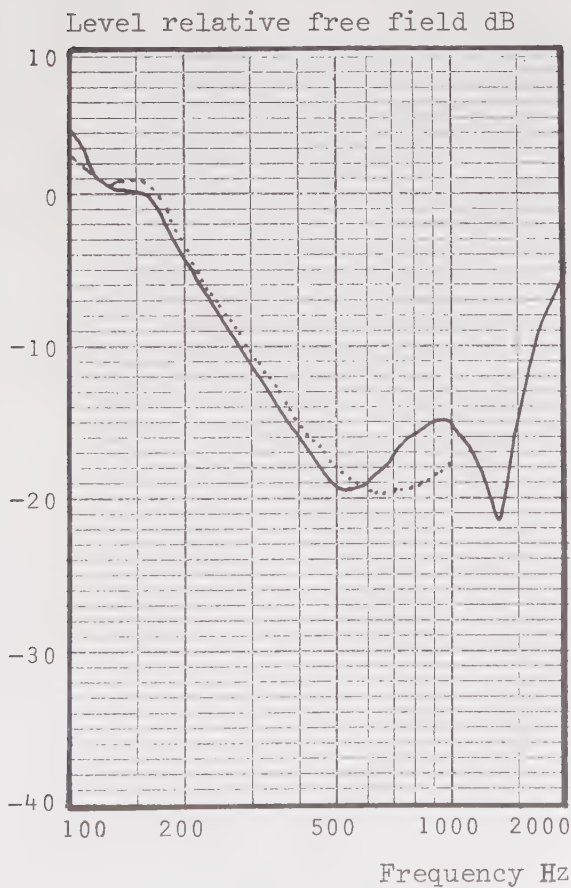
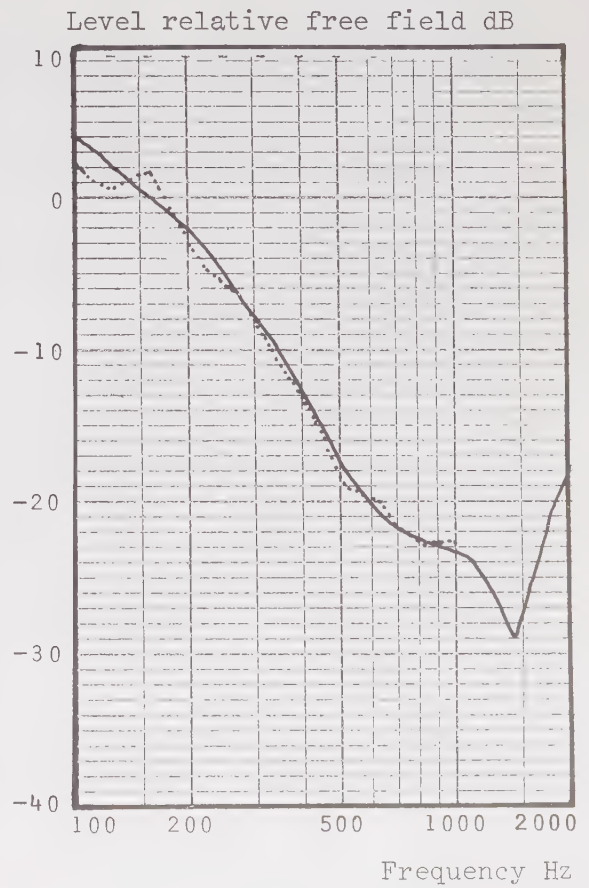
$$(x_R, y_R, z_R) = (-40, 0, z_R) \text{ m}$$

FIG.356. $z_R = 2.0 \text{ m}$ FIG.357. $z_R = 1.0 \text{ m}$ FIG.358. $z_R = 0.5 \text{ m}$ FIG.359. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (14, -14, 0.1) \text{ m}$$

$$(x_R, y_R, z_R) = (-14, 14, z_R) \text{ m}$$

FIG.360. $z_R = 2.0 \text{ m}$ FIG.361. $z_R = 1.0 \text{ m}$ FIG.362. $z_R = 0.5 \text{ m}$ FIG.363. $z_R = 0.1 \text{ m}$

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (10, 0, 0.1) \text{ m} \quad (x_R, y_R, z_R) = (-10, 0, z_R) \text{ m}$$

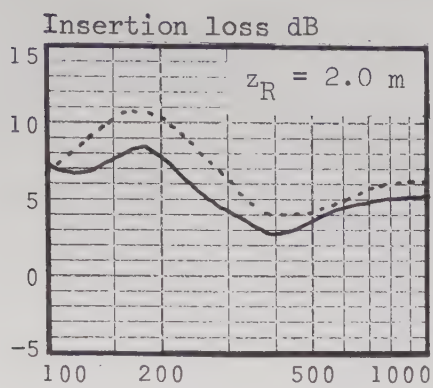


FIG.364. (Figs.319 & 336)

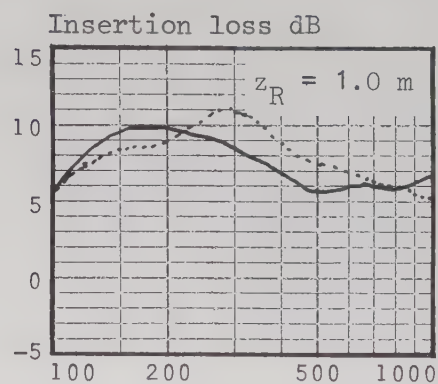


FIG.365. (Figs.320 & 337)

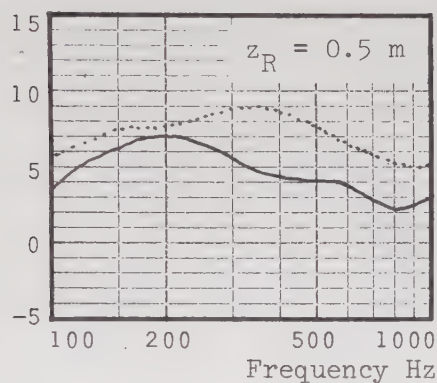


FIG.366. (Figs.321 & 338)

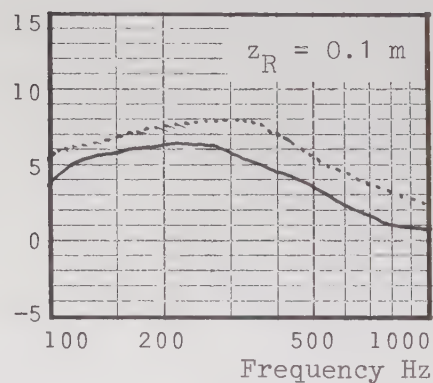


FIG.367. (Figs.323 & 339)

$$(x_S, y_S, z_S) = (20, 0, 0.1) \text{ m} \quad (x_R, y_R, z_R) = (-40, 0, z_R) \text{ m}$$

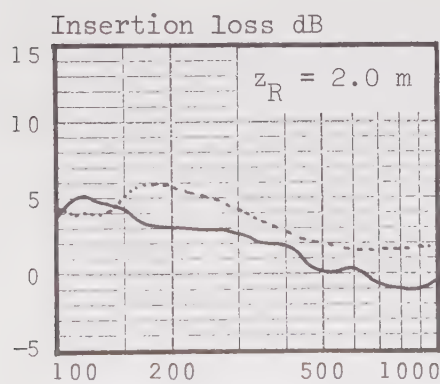


FIG.368. (Figs.331 & 356)



FIG.369. (Figs.332 & 357)

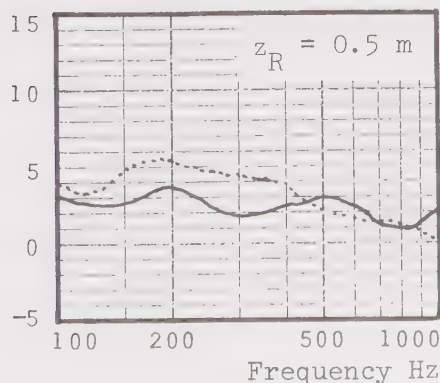


FIG.370. (Figs.333 & 358)

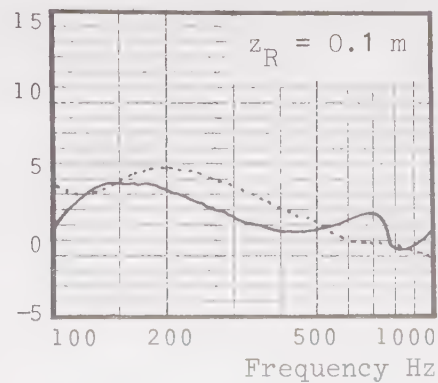


FIG.371. (Figs.335 & 359)

$$(y_0, y_1, z_1) = (-12.5, 12.5, 1.5) \text{ m}$$

$$(x_S, y_S, z_S) = (20, 0, 0.1) \text{ m} \quad (x_R, y_R, z_R) = (-20, 0, z_R) \text{ m}$$

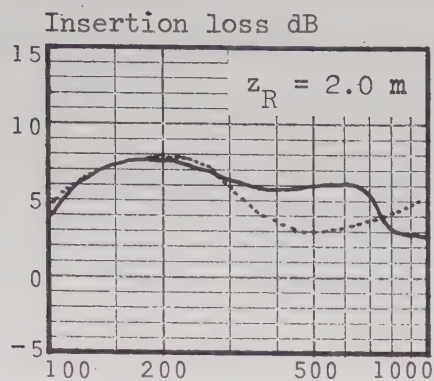


FIG.372. (Figs.325 & 352)

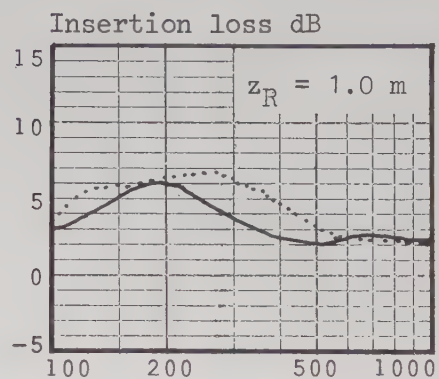


FIG.373. (Figs.326 & 353)

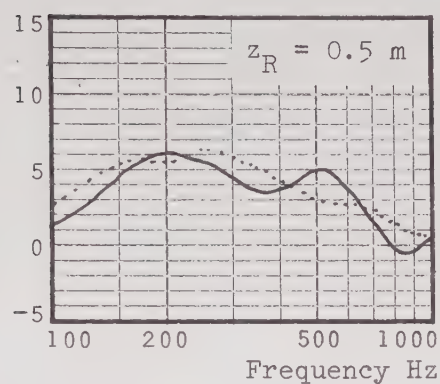


FIG.374. (Figs.327 & 354)

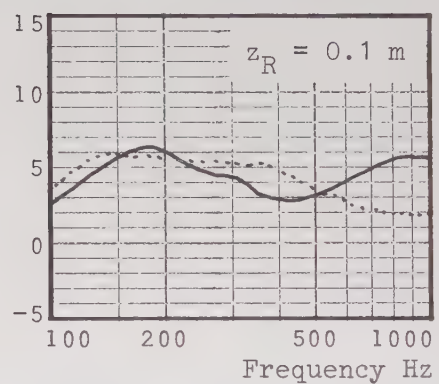


FIG.375. (Figs.329 & 355)

$$(x_S, y_S, z_S) = (14, -14, 0.1) \text{ m} \quad (x_R, y_R, z_R) = (-14, 14, z_R) \text{ m}$$

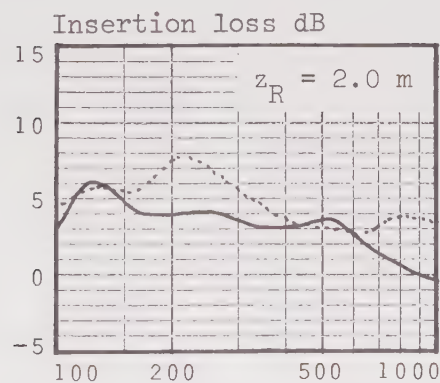


FIG.376. (Figs.325 & 360)

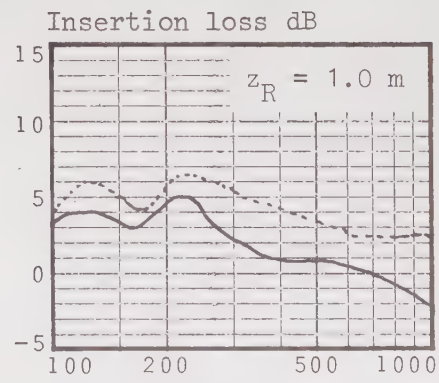


FIG.377. (Figs.326 & 361)

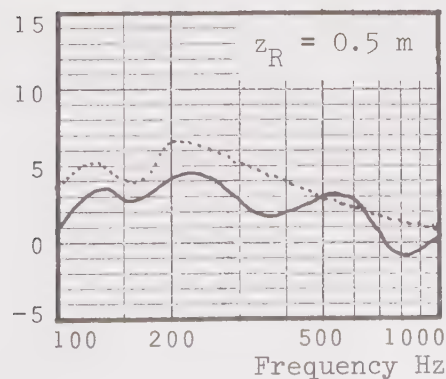


FIG.378. (Figs.327 & 362)

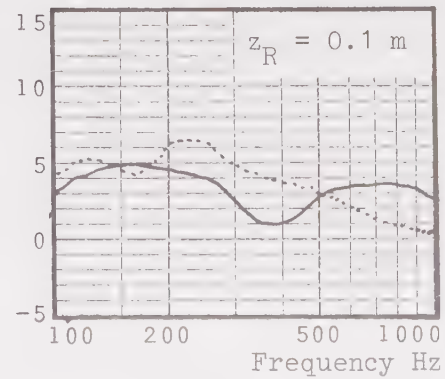


FIG.379. (Figs.329 & 363)

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11. Reference 2, Sec. IV:B.
12. Reference 2, Sec. IV:D.

